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Theory, Numerics, Applications

Abstracts

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July 25, 1994

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Dear Dr. Malek-Madani:

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Sincerely,

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On Genuinely Multi-dimensional Upwinding in the Finite Volume Context

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Genuinely multidimensional upwinding is a very active research field since a few years. The goal is to improve the now classical multidimensional upwind schemes (that rely on 1-D Riemann solvers) by a better treatment of of the multidimensional features of the flow. One may cite [1] in the finite volume context where a tricky modification of 1-D Riemann solvers is used. Nevertheless, the most advertised way to achieve this goal seems to be the so called fluctuation schemes by Roe and Deconinck (see [2] for a review and [3] for the state of the art). These schemes rely on three steps : a genuinely multidimensional second order scheme, a Roe type linearization of the Euler equations and a wave model. The most criticized step of this method is the last one because the choice a specific wave model is not a clear task. As we will see it later, the second step can also be criticized.

Here, we have chosen to follow the Roe's original methodology : find a linearization consistent with the Euler equation and then solve the Riemann problem for the linearized equation to define a numerical flux.

A consistent linearization can be constructed as follows. In the plane, we consider n angular sectors $D_i, i = 1, \cdots, n$, that meet at point O ; a constant state W_i is assumed in each D_i. Let us denote by $W^{(0)}$ this initial condition. For the sake of simplicity, we assume three angular sectors, but the generalization is easy. We also assume the uniqueness of the (self-similar) solution in L^∞, denote by $\mathcal{R}(x/t, y/t, W^{(i)})$: the behavior of the 2-D Riemann problem, even in a simplier geometrical situation, is a very difficult problem [5]. This assumption is well supported by all the numerical simulations. For any triangle T which intersection with the D_i's is never empty and for any real number λ, we define T_λ the triangle obtained from T by $T_\lambda = O + \lambda\, T$. The divergence theorem applied to T_λ gives

$$\lim_{\lambda \to +\infty} \frac{1}{\lambda}\left(\int_{T_\lambda} \left[\mathcal{R}(\xi,\nu,W^{(i)}) - W^{(i)}(\xi,\nu)\right] d\xi\, d\nu\right) + \frac{1}{2}\int_T F_{\vec{n}}(W^{(0)})dl = 0 \qquad (1)$$

where as usual, $\vec{n}$ is the outward normal unit vector and $F_{\vec{n}}$ the normal flux to the boundary of T. If $W_t + \overline{A}W_x + \overline{B}W_y = 0$ is a linearization of the Euler equations, a similar equation as (1) can be obtained between the exact solution of the linearized problem with the initial condition $W^{(0)}$. A requirement on $\overline{A}$ and $\overline{B}$ can be that in the limit $\lambda \to +\infty$, the exact solutions of both Riemann problems are equal in average. This leads to the equation :

$$\sum_{i=1,3} \overline{A}_{\vec{N}_i} . W_i = \sum_{i=1,3} F_{\vec{N}_i}(W_i) \qquad (2)$$

where $\vec{N}_i = \int_T \chi_i \vec{n} dl$ (χ_i is the characteristic function of D_i). In 1-D, this condition on the linearized Jacobian matrix leads to the well known Roe average [4].

In the present case, we assume a perfect gas equation of state with a constant ratio of specific heats, γ, and the following condition on $\overline{A}$ and $\overline{B}$:

$$\overline{A} = \frac{\partial F}{\partial W}\left(\overline{W}(W^{(0)})\right) \qquad \overline{B} = \frac{\partial G}{\partial W}\left(\overline{W}(W^{(0)})\right), \qquad (3)$$

where $\overline{W}(W^{(0)})$ is an average state to determine. This assumption simplifies the system (2) : no equation on the density, two quadratic equations on the x- and y- velocity involving *only* the velocity and a cubic equation on the total enthalpy.

The two equations on the component of the velocity can be interpreted geometrically as the intersection of two hyperbolas. There are always at least two solutions to the problem, and possibly four [a]. In the full paper, we provide a parametric study of the solutions and describe a criterion to get the "right" solution of the problem [7]. We also discuss the hyperbolicity of the linearized system i.e. the positivity of the square of the averaged speed of sound.

The Roe's, Struijs & Deconinck linearization does not satisfy condition (2) , and then *is not consistent with the Euler equations*. A simple counter example is given by two states W_L and W_R separated by a stationary planar shock. We artificially divide the left half plane in two parts with the same state, W_L. Roe's et al solution is does not give the true jump conditions contrarily to ours that reduces to the classical 1-D jump conditions and then to Roe's original equation. Nevertheless, their linearization procedure always lead to a hyperbolic system.

Once this is done, it remains to solve the linearized problem exactly. One first notices that a Galilean change of variables reduces the problem to the solution of a pure acoustic one. The main difficulty of this latter one lies in the computation of the solution in the vicinity of point O, more precisely in a disc of center O and radius $\overline{c} = \sqrt{(\gamma - 1)(\overline{H} - \frac{1}{2}[\overline{u}^2 + \overline{v}^2)]}$ ($\overline{H}$ is the enthaipy and $\overline{u}$, $\overline{v}$ are the components of the velocity of the averaged state $\overline{W}(W^{(0)})$). To get the solution in this disc, the idea is to rewrite it in term of the eigenvectors of $\cos\theta\overline{A} + \sin\theta\overline{B}$, for any θ, thus the independent variables are now θ and $r = \sqrt{\xi^2 + \nu^2}$, and a change of variable leads to a Laplace equation that is solvable because the boundary condition on the circle of radius $\overline{c}$ are deduced from the Rankine-Hugoniot relations for the linearized equation. Fortunately, it is easy to write them in the r-, θ- coordinates and then to write down an explicit solution. Away for this disc, the solution reduces to the classical 1-D one. All the details can be found in [6].

Then a numerical flux can be constructed. For now, no experimental studies has been conducted to validate this method but we hope to make some for the time of the conference.

[1] D.W. Levy, K.G. Powell, and B. van Leer, "Use of a Rotated Riemann Solver for the 2-D Euler Equations," *J. Comput. Phys.*, vol. 106, 1993.

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[3] R. Struijs, H. Deconinck, and P.L. Roe, "Fluctuation Splitting Schemes for the 2D Euler Equations," Von Karmàn Institute for Fluid Dynamics Lecture Series 1991-01, February 18–22, 1991.

[4] P.L. Roe, *J. Comput. Phys.*, vol. 63, 1981.

[5] T. Zhang and Y. Zheng, "Conjecture on the Structure of Solutions of the Riemann Problem for Two-Dimensional Gas Dynamic Systems," *SIAM J. Math. Anal.*, vol. 21, pp. 593–630, 1990.

[6] R. Abgrall, "A Genuinely Multidimensional Riemann Solver," INRIA Report No. 1859, February, 1989.

[7] R. Abgrall, "A Consistent Linearization for the 2-D Euler Equations," INRIA Report, in preparation.

[a] In the 1-D case a similar approach can be adopted. There are always two solutions : the classical Roe's solution and a spurious one that is eliminated because it becomes unbounded in the limit of a vanishing density gradient

A Method with Automatic Error Control for Numerical Solution of the Wave Equation

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A numerical method producing lower and upper bounds for the solution of the wave equation is presented. We focus our attention on the one-dimensional equation

$$u_{tt} - u_{xx} = f(x, t, u) \tag{1}$$

as a model example but most of the results can be generalized for the multidimensional case.
The development of the method is based on the concept of Operator of Monotone Type as introduced by L.Collstz. We use that under certain conditions the hyperbolic operator in (1) is an operator of monotone type.
The method is of spectral type. The bounds are obtained in the form of trignometric polynomials about x and quadratic splines about t using nodes $t_k, k = 0, 1, 2, \ldots$ In the nonlinear case an iterative procedure producing contracting bounds is applied in any time interval.
This numerical solution carries within itself an assurance of its quality because the exact solution is enclosed by a lower and an upper bound. However a theoretical estimate of the rate of approximation is also presented.
Numerical results for some linear and nonlinear ($f(x, t, u) = ku^3 + \phi(t, x)$) equations are considered.
Application of Interval and Computer Arithmetic to secure the bounds against rounding errors and to obtain higher accuracy is also discussed.

Two-Dimensional Finite Volume Extension of the Nessyahu-Tadmor Scheme for Conservation Laws

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Presented by Paul Arminjon

In an attempt to construct a simplified version of high resolution non-oscillatory Godunov-type methods for hyperbolic conservation laws, Nessyahu and Tadmor proposed an elegant scheme based on a combination of the staggered form of the Lax-Friedrichs scheme and van Leer's high resolution Godunov-type methods. The main feature of their scheme lies in the fact that it does not require the detailed solution of the Riemann problems generated at the cell interfaces, thanks to the use of the staggered Lax-Friedrichs scheme, and still gives second order resolution.
We present a two-dimensional finite volume method based on applying the principle of their scheme to a finite element triangulation and a finite volume formulation using the barycentric cells as well as a dual set of cells to perform the second step of the scheme.
In a separate paper, we prove an L^∞ estimate of the numerical solution and obtain L^∞-weak convergence to a weak solution of $u_t + \vec{V} \cdot grad\, u = 0$. The method is presently applied to several test problems.

[a]This research was supported by a grant from the National Science and Engineering Research Council of Canada, and by an FCAR grant from the Government of the Province of Québec.

The Initial Boundary Value Problem for the Hyperbolic System of Conservation Laws

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Let us consider a general n-system of conservation laws:

$$U_t + F(U)_x = 0. \tag{1}$$

We assume that (1) is hyperbolic with *non-zero characteristic speeds* and each characteristic field is *genuinely nonlinear* in the sense of Lax.
We study the large-time behavior of solutions of the initial boundary value problem of (1) with prescribed initial values:

$$U(x,0) = U_0(x) \qquad \text{for} \qquad x > 0 \tag{2}$$

and boundary conditions:

$$F(U(0,t)) - H(t) \in \mathcal{M} \qquad \text{for} \qquad t > 0 \tag{3}$$

Here $U_0(x)$ and $H(t)$ aare given vector functions and $\mathcal{M}$ is a fixed subspace of R^n whose dimension is $n - n_+$ (n_+ : the number of the positive eigenvalues of $F'(U)$). Let $\mathcal{E}^+(U)$ be the direct sum of the eigenspaces corresponding to the positive eigenvalues of $F'(U)$. We further assume that the boundary conditions satisfy *Lopatinski* condition for all $U \in \Omega$:

$$\mathcal{E}^+(U) \in \mathcal{M} = 0. \tag{4}$$

We can show that, as $t \to \infty$, the solution converges to the superposition of shock waves and rarefaction waves whose strength and speed depend only on the data at infinity. We also obtain algebraic rates of convergence. Proof consists in showing that total amount of interaction of waves involving reflected waves is uniformly bounded.

Singularities and Interfacial Patterns in Hele-Shaw Flows

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Interest in interfacial motion in Hele-Shaw cells stems from analogies with flow in porous media, dendritic growth, and bubble competition in Rayleigh-Taylor instabilities. Depending on the strength of surface tension effects, interfacial patterns may contain multiple fingers in competition, tip-splitting at the leading edge of a finger, branching along the sides of a finger, and even fractal behavior for very small surface tension coefficients. The usual fluid equations for the flow can be simplified by a conformal map of the physical domain into a unit disk. The presence of singularities in the conformal map outside the disk introduce specific structures in the physical domain. Depending on the location and nature of the singularities we find interfacial patterns such as fingers undergoing competition, tip-splitting and side-branching. Generically, singularities move towards the boundary of the unit disk, inducing the formation of these interfacial patterns. We present a new numerical method that can track the location of singularities outside the unit disk in time efficiently. This allows us to watch the development of interfacial patterns as singularities approach the unit disk.

Wave Processes in a Three Phase System of a Collapsing Bubble Near a Compliant Wall

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Presented by Josef Ballmann

The material damaging process caused by cavitation bubbles and characterized by strong shock waves and liquid jets towards the wall is still an unsolved problem. The shock waves arise first and foremost in the liquid phase [1] which is therefore to be assumed as compressible. So the three phases are a compressible viscous gas phase in the bubble, a compressible viscous liquid phase and the compliant wall which is assumed to be elastic-plastic. Neglecting the viscous effects in the fluid the conservation laws in the three phases form three different hyperbolic systems which are coupled at the material interfaces by jump conditions.

The paper deals with the basic problem of a single bubble collapsing near a compliant wall, including the viscous effects. A second order FEM-FCT method is developed to treat the two fluid phases in a fully coupled way whereby the conditions at the gas liquid interface (surface traction, heat transfer and condensation/evaporation) are integrated into the solution scheme like for an interior boundary. The method contains two time steps where the diffusive effects are taken into account in the second step. At the fluid solid interface the method is coupled with a two step Godunov type method for elastic-plastic waves in solids [2]. The jump condition at the fluid solid interface are satisfied applying both methods iteratively.

The paper will present the solution method and numerical results for a typical application.

[1] H. Westenberger, "Zur Dynamik des Impulstransportes in Blasigen, Kompressiblen Flüssigkeiten," Dr. Rer. Nat. Thesis, RWTH Aachen 1987.

[2] X. Lin and J. Ballman, "A Riemann Solver and a Second-Order Godunov Method for Elastic-Plastic Wave Propagation in Solids," *Int. J. Impact Engrg.*, vol. 13, pp. 463–478, 1993.

Multidimensional Algorithms for Relativistic Hydrodynamics

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In this work we present a formulation of the Riemann solver for relativistic hydrodynamics. The Riemann solver presented here is an extension to the relativistic regeime of the high quality, two shock Riemann solver for non-relativistic hydrodynamics presented by Van Leer (1979) and Colella (1982). It, therefore, shows the same advantages that the previously mentioned Riemann solvers show for non-relativistic hydrodynamics, viz. exact treatment for strong shocks and accurate treatment of contact discontinuities. We then incorporate the Riemann solver into a multidimensional TVD algorithm and show that the algorithm allows to calculate with precision several flows with very high Lorenz factors (i.e. flows with speeds very close to the speed of light).

Roe-Type Riemann Solver for Magnetohydrodynamics

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Presented by Dinshaw S. Balsara

In this paper we present linearized formulations of the Riemann problem for magnetohydrodynamics (MHD). Both algebraic and canonical path formulations have been constructed. Numerical experiments show the difference in the formulations to be slight. The approximate Riemann solver has Roe's "Property U" ensuring that isolated shocks are captured exactly. The Riemann solver has been used in a sequence of multidimensional upwind TVD schemes. The schemes have exceptional entropy enforcement and very high resolution. Several details of the schemes are presented here.

Eulerian Formulation of the MHD Riemann Problem

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In this paper we formulate the Riemann problems for non-relativistic and relativistic MHD using the technique of Lorenz transformations developed by Balsara (1994) J.Comp.Phys. This permits us to arrive at an exact Riemann solver. Moreover, the problem resolves itself into its component waves in the eulerian frame. This allows schemes for MHD that are formulated in the direct eulerian frame of reference to make use of this Riemann solver. Results are presented for this formulation. Comparisons with a Roe formulation for the same problem are also presented.

A Zero-Relaxation Limit Model for Two-Phase Fluid Flows

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An important problem affecting solid rocket motors is the existence of combustion instabilities. These are oscillations developing in the motor which are coupled with the acoustic modes of the motor and which can be very damageful. In modern solid rocket motors, small aluminium particles are included in the propellant to increase the specific impulse. Their presence also affects the flow inside the motor and can attenuate these instabilities (see [1]). We are interested in the numerical prediction of the damping.

To simplify the analysis, we consider that all the particles in the flow have the same radius r, and that the gas and the particles only interact through the drag force. Thus the flow inside the motor can be modelled as a two-phase fluid flow, one phase consisting in the burnt gases, the other one consisting in the cloud of oxidated aluminium particles (see [2]). Under reasonable assumptions, this model writes as an hyperbolic system of conservation laws with a stiff relaxation term:

$$\partial_t \mathbf{U} + \partial_x \mathbf{F}(\mathbf{U}) = \frac{\mathbf{R}(\mathbf{U})}{\epsilon}, \mathbf{U} \in \mathbb{R}^6. \tag{1}$$

The relaxation time ϵ is proportional to the square of the radius r.

Since the source term is stiff, we expect (see [3]) system 1 to behave like its zero-relaxation limit for small size particles.

We derive the zero-relaxation limit and give its order 1 correction (following [4]), which leads to an hyperbolic system with new variables $\mathbf{u} \in \mathbb{R}^5$ which writes:

$$\partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = \epsilon \partial_x (\mathbf{D}(\mathbf{u}) \partial_x \mathbf{u}), \tag{2}$$

where $\mathbf{D}(\mathbf{u})$ is a non-negative tensor.

Numerically, we solve both system using Strang operator splitting combined with a second order Van Leer scheme. We impose the timestep to be determined solely by the CFL condition on the hyperbolic part. We compare both system on the prediction of the damping of a sine wave propagating in the flow. As expected, when the relaxation time is small compared to the timestep, system (2) predicts the correct attenuation, while system (1) can not.

But we show that an important feature of the physical system, namely the existence of an optimal radius for the attenuation is not predicted by the relaxed system (2) even with its first-order correction. We give a validity criterion on the relaxation time to determine when the relaxation limit looses its accuracy.

[1] F.E. Cullick, "Combustion Instability in Solid Rocket Motors," CPIA Publication 290, 1981.

[2] L. Sainsaulieu, Ph.D Thesis, Ecole Polytechnique, 1991.

[3] T.-P. Liu, "Hyperbolic Conservation Laws with Relaxation," *Commun. Math. Phys.*, vol. 108, 1987.

[4] G.-Q. Chen, D. Levermore, T.-P. Liu, "Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy," preprint, 1992.

Cartesian Grid Algorithms for 3-D Euler Flows in Complex Geometry

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We discuss Cartesian (non-body-fitted) mesh algorithms for solving fluid flow problems in complicated geometries. Cartesian mesh methods have the advantage that no explicit mesh generation or body fitted grids are needed, which greatly reduces the human effort needed for complex flow computations. Also, Cartesian grid difference methods, used in the interior of the flow region, are accurate, robust, and vectorizable. However, it is a challenge to find stable and accurate difference formulas for the irregular cells cut by the boundary. We present our approaches to this difficult problem. We give numerical convergence results for test problems in 2D steady flow. Computational results for full 3D aircraft are also presented.

Development of Singularities of Solutions of a Nonlinear Hyperbolic Problem Despite Boundary Damping

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Presented by Pascale Bergeret

Let us consider a strictly hyperbolic system of two first order partial differential equations written in the diagonal form

$$\begin{cases} \partial_t w_1 + \lambda_1(w)\,\partial_x w_1 = 0 \\ \partial_t w_2 + \lambda_2(w)\,\partial_x w_2 = 0 \end{cases} ,t > 0, \lambda_1(w) < 0 < \lambda_2(w) \forall w, \tag{1}$$

where w is a vector with two components w_1, w_2.
The initial value w^0 is assumed to be closed to a constant $\bar{w}$, which we can take equal to 0.
It is well known that smooth solutions of (1) generally exist only for a finite time, breaking down because their first derivatives blow up. The time beyond which a solution cannot be continued can be given asymptotically (see P.D. Lax).
When considering the same system in the strip $\mathbb{R}^+ \times]0, L[$, one adds boundary conditions

$$w_2 = f(w_1) \text{ at } x = 0 \quad \text{and} \quad w_1 = g(w_2) \text{at } x = L.$$

Compatibility conditions will insure at least the local existence of a C^∞ solution.
J.M. Greenberg and Li Ta-tsien proved there exists a unique global solution whenever the initial data is small in the C^∞ norm and $|f'(0)\, g'(0)| < 1$.
The question to be discussed is the importance of the smallness in the C^∞ norm postulated in the theorem above : it will be shown that there exist initial data which are small in the C^1 norm and whose derivatives blow up after several reflexions despite the boundary damping $|f'(0)\, g'(0)| < 1$.

[1] J.M. Greenberg and Li Ta-tsien, "The Effect of Boundary Damping for the Quasilinear Wave Equation," *J. Differential Equations*, vol. 52, pp. 66–75, 1984.

[2] P.D. Lax, "Development of Singularities of Solutions of Nonlinear Hyperbolic Partial Differential Equations," *J. Math. Phys.*, vol. 5, pp. 611–613, 1964.

The Numerical Wave Speed for Scalar Hyperbolic Conservation Laws with Source Terms

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Presented by A.C. Berkenbosch

We study scalar hyperbolic conservation laws with source terms in one space dimension, i.e.

$$\frac{\partial}{\partial t}u(x,t) + \frac{\partial}{\partial x}f(u(x,t)) = q(u(x,t)), \;\; \forall x \in \mathbb{R}, \; \forall t > 0. \tag{1}$$

Conservation laws of this form occur, among others, in the theory of reacting gas flow [1]. In this case, the term $q(u)$ describes the production or consumption of the variable u, per unit length and per unit time. We assume that $u = 0$ for the unreacted gas and $u = 1$ for the completely reacted gas. A typical form for q, which is often used, is the ignition model:

$$q(u) = \begin{cases} 0 & \text{if } u \le u_{ign}, \\ > 0 & \text{if } u > u_{ign}, \end{cases} \tag{2}$$

where $0 \le u_{ign} < 1$. Hence, the reaction starts as soon as $u > u_{ign}$ (the ignition value).
A very natural way to solve (1) numerically is by a splitting method, in which the numerical solution is computed in two steps. In the first step the homogeneous conservation law $u_t + f_x = 0$ is solved with a conservative difference method. In the second step we solve the ordinary differential equation $u_t = q(u)$ for the chemical reaction. For stability reasons the second step is performed implicitly.
When (1) is solved numerically, discontinuities are observed at wrong locations, i.e. the numerical solution is propagating with non-physical speeds [2]. For the splitting method it can be proved that for fast reactions

$$S = s + \frac{\Delta x}{\Delta t}(1 - C), \tag{3}$$

where S is the numerical speed, s is the exact speed, Δt is the time step, Δx is the mesh size and C is a constant with $u_{ign} < C \le 1$. If $u_{ign} = 0$, then it can be shown that (3) implies $S = \Delta x/\Delta t$. Hence, the numerical solution is propagating with the non-physical speed of one grid point per time step [1,2].
If we compare the ignition model (2) with the physical more realistic Arrhenius' model it follows that we should take $u_{ign} > 0$. In that case the numerical speed is closer to the exact speed, since $S \downarrow s$ as $u_{ign} \uparrow 1$. However, it follows from (3) that the numerical solution propagates too fast for all $u_{ign} < 1$.

[1] P. Colella, A. Majda and V. Roytburd, "Theoretical and Numerical Structure for Reacting Shock Waves," *SIAM J. Sci. Stat. Comput.*, vol. 7, pp. 1059–1080, 1986.

[2] R.J. LeVeque and H.C. Yee, "A Study of Numerical Methods for Hyperbolic Conservation Laws with Stiff Source Terms," *J. Comput. Phys.*, vol. 86, pp. 187–210, 1990

Implementation of a High Order 2-D ENO Scheme [a]

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Presented by Barna L. Bihari

We present a high order *essentially nonoscillatory* (ENO) scheme for cartesian coordinates, which is implemented using two dimensional multiresolution analysis. The multiresolution representation is used to perform an analysis of the regularity of the solution. As for most conservation law problems the solution is overresolved in some or much of the computational domain, there is no need to use the costly ENO reconstruction everywhere. The multiresolution analysis serves as a set of sensors which allows us to selectively use the ENO interpolation only where it is needed.

It is well known that nonlinear hyperbolic conservation laws admit discontinuous solutions, which then makes it necessary to use a nonoscillatory interpolation. Our implementation combines ideas introduced in [1] for an ENO reconstruction with the method of multiresolution analysis for improving efficiency (first presented for 1-D in [2]). The prototype conservation laws will be solved using continuous and discontinuous initial data and arbitrarily high order ENO reconstructions. For smooth regions, the semi discrete formulation makes it natural to interpolate the right hand side in a multiresolution sense. For cells near a discontinuity the fluxes are directly computed, and so is the right hand side. We hope to show that the efficiency of the solution procedure can thus be improved by as much as 10 to 20 times, while preserving the quality of the solution itself.

The speaker will present analysis and computational results for multiresolution ENO schemes of various orders of spatial accuracy.

[1] A. Harten and S.R. Chakravarthy, "Multidimensional ENO Schemes for General Geometries," UCLA CAM Report 91-16, August, 1991.

[2] A. Harten, "Multiresolution Algorithms for the Numerical Solutions of Hyperbolic Conservation Laws," UCLA CAM Report 93-03, March, 1993; to appear in *Commun. Pure Appl. Math.*

[a] Research supported by Rockwell International IR&D funds.

A Study of Stability of Strong Discontinuities and Hyperbolic Problems

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The motion of continuum is known to occur commonly with surfaces of strong discontinuity present (naturally, under a certain idealization of physical processes) which separate the domains of parameter continuity and characterize such motion. This report, based on a large number of works performed by author and his colleagues, presents some conceptual reasoning which concerns the mathematical theory of strong discontinuities in continuum mechanics. More precisely, we touch upon one of the most important aspects of the theory —the topic of studying the well-posedness of mixed problems on stability of strong discontinuities in continuum mechanics.

It should be also pointed out that the problem of stability of strong discontinuities in continuum mechanics has assumed a special significance lately in view of the wide application of methods of mathematical modelling, and, in particular, computational methods for finding approximate solutions to the problems of continuum mechanics with strong discontinuities. Clearly, the computational algorithms for finding motions of continuum with strong discontinuities can be applied only if the topic on stability of discontinuities in a given model is elucidated to the end.

Applications of Characteristic Theory to 3-D Computational Fluid Dynamics Problems

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Let $Q(\mathbf{r})$ be the given power of the energy sources distributed in the space with a radius vector $\mathbf{r}$. In the case of steady flow of inviscid non-heatconducting gas, the system of gasdynamic equations is equivalent to the five scalar partial differential equations for five unknown scalar functions, viz., three components of the velocity vector $\mathbf{V}(\mathbf{r})$, entropy $s(\mathbf{r})$ and the total energy $H(\mathbf{r})$. Naturally, the number five also defines the order of the system. Suppose that we are given a smooth surface with a unit normal vector $\mathbf{n}(\mathbf{r})$. Let $\mathbf{e}$ be a constant free unit vector. Suppose that $\mathbf{e} \cdot \mathbf{n} \neq 0$ at a certain part of the smooth surface. For a unique solvability of the Cauchy problem with the data specified at the surface with a normal $\mathbf{n}$, we must determine from initial system five outgoing derivatives, defined by the expressions $(\mathbf{e} \cdot \nabla)\mathbf{V}$, $\mathbf{e} \cdot \nabla s$, $\mathbf{e} \cdot \nabla H$. The characteristic properties of the initial system are well known. On the characteristic flow surfaces, the normal characteristic vector satisfies the equation $\mathbf{V} \cdot \mathbf{n} = 0$ and is a triple root of the characteristic equation. Hence, three compatibility relations containing the internal derivatives of the basic unknown functions are obtained for these surfaces. The remaining two equations of the initial system are also compatibility relations, but are used for determination the outgoing derivatives of the basic unknown functions on the flow surface. This fact is generally ignored during an analysis of characteristic properties of the initial system. However, the remaining compatibility relations can be used to find out the outgoing derivatives from the flow surface which are determined by the data on the surface and those, which must be specified for a unique solvability of the Cachy problem. On the characteristic wave surfaces, the characteristic normal vector satisfies the condition $\mathbf{M} \cdot \mathbf{n} = 1$ and is a singular root of the characteristic equation. Hence, one compatibility relation containing the internal derivatives of the basic unknown functions is obtained for these surfaces. Four compatibility relations with outgoing derivatives are defined on these surfaces. The value $\mathbf{n} \cdot (\mathbf{e} \cdot \nabla)\mathbf{V}$ must be specified.

The complete system of compatibility relations is used for constructing numerical schemes for the method of spatial characteristics on the wave surfaces of opposite families. In a general case along the line of intersection the ten functions $\mathbf{V}$, s, H; $(\mathbf{e} \cdot \nabla)\mathbf{V}$, $\mathbf{e} \cdot \nabla s$, $\mathbf{e} \cdot \nabla H$ can be found by using opposite families of five compatibility relations each. If the surface are not characteristic and fixed we have pseudocharacteristic schemes. Using a characteristic algorithm we can compute the supersonic flow with error a few tenths of a percent and faster than the marching methods or time-dependent stationing methods. This paper is a development of P.I.Chushkin's method.

Examples of computations are given for the following problems: the calculations of 3-D supersonic flows with the energy sources, the solar wind flow in the Earth's magnetosphere (the system of the eighth order), shape optimization of supersonic parts of 3-D nozzles.

Viscosity Solutions and Uniqueness for Systems of Conservation Laws

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The talk will present a new method, based on wave-front tracking, for the construction of approximate solutions to a 2×2 system of conservation laws:

$$u_t + \Big[F(u)\Big]_x = 0, \qquad u(0,x) = \bar{u}(x). \qquad (*)$$

As usual, we assume that the system is strictly hyperbolic, and that each characteristic field is genuinely nonlinear or linearly degenerate. For every $\bar{u} \in \mathbf{L}^1$ with small total variation, the approximating sequence is Cauchy and converges to a unique limit solution, depending continuously on the initial data.

This algorithm determines a Lipschitz continuous semigroup $S : [0,\infty) \times \mathcal{D} \mapsto \mathcal{D}$, on a domain $\mathcal{D} \subset \mathbf{L}^1$, with the following property. If $\bar{u}$ is piecewise constant, then, for $t > 0$ small enough, $S_t\bar{u}$ coincides with the solution of (*) obtained by piecing together the corresponding self-similar solutions of the corresponding Riemann problems.

For general $n \times n$ systems, a semigroup with the above property will be called a *Standard Riemann Semigroup* (SRS). If it exists, a SRS is necessarily unique. Its trajectories can be characterized as being *viscosity solutions* of the system (*), according to a definition which will be here introduced.

One can prove that every limit of approximate solutions constructed (with probability one) by the random scheme of Glimm, or by the deterministic version of Liu, or by any wave-front tracking method, is indeed a viscosity solution. Hence, if a SRS exists (as it is certainly the case for 2×2 systems), all these algorithms produce a unique solution, depending Lipschitz continuously on the initial data.

A von Neumann Reflection for the 2-D Burgers Equation

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Presented by M. Brio

The two-dimensional Burgers equation gives an asymptotic description of the transition from regular to irregular reflection for weak shocks. We present numerical solutions of the two-dimensional Burgers equation which contain a von Neumann reflection. These solutions provide independent confirmation of the von Neumann reflection discovered by Colella and Henderson in numerical solutions of weak shock reflection using the full compressible Euler equations. The two-dimensional Burgers problem is nonstandard because it is a characteristic initial value problem. We discuss the stability and convergence of the numerical scheme.

Solutions with Shocks in Several Variables

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We consider autonomous systems of the form

$$\sum_{j=1}^{l}\sum_{i=1}^{n} a_{ijs}(u)\partial_{x_i}u_j = 0, \quad s = 1, r \geq l \tag{1}$$

These systems are such that all Jacobi matrices $D(u) = (\partial_{x_i}u_j)$ satisfying the system, are of the form

$$Du = \sum_{i=1}^{q} \lambda^{(i)}(u) \otimes \gamma^{(i)}(u), \quad q < \infty, \tag{2}$$

where for $\lambda^{(i)}(u) = (\lambda_1, \ldots, \lambda_n)$, $\gamma^{(i)} = (\gamma_1, \ldots, \gamma_l)$ the condition $\sum_{i=1}^{n}\sum_{j=1}^{l} a_{ijs}(u)\lambda_i\gamma_j = 0$, $s = 1, \ldots, r$ is fulfilled Hyperbolic systems represent the most important example of this kind of systems.
The property (2) allows the construction of the solutions u $R^n \supset D \to R^l$ in two steps. First we construct the image $u(D) \subset R^l$ of the required solution, then we parametrize $u(D)$ by the independent variables $x_l, \ldots, x_n$. Our goal is to show, that we may construct the image $u(D)$ with some appropriate singularities, so that after enough simple parametrization of $u(D)$ by $x_1, \ldots, x_n$ we obtain solutions admitting the prescribed system of interacting shocks. For example for the system $\partial_t c + v_1\partial_{x_2}c + v_2\partial_{x_2}c + kc\operatorname{div}v = 0$, $\partial_t v_i + v_1\partial_{x_2}v_i + v_2\partial_{x_2}v_2 + c.k\partial_{x_i}c = 0, i = 1, 2$, we obtain exact solutions giving two films of the following form where $t_1 < t_2 < t_3 < t_4$ See Figure 1.

Hybrid High Order Methods for Two Dimensional Detonation Waves

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In order to study multi-dimensional unstable detonation waves, we have developed a high order numerical scheme suitable for calculating the detailed transverse wave structures of multidimensional detonation waves. The numerical algorithm uses a multi-domain approach so different numerical techniques can be applied for different components of detonation waves. The detonation waves are assumed to undergo an irreversible, unimolecular reaction $A \rightarrow B$. Several cases of unstable two dimensional detonation waves are simulated and detailed transverse wave interactions are documented. The numerical results show the importance of resolving the detonation front without excessive numerical viscosity in order to obtain the correct cellular patterns.

[a] Research supported has been funded by NSF ASC ASC 9113895.

Free-Boundary Value Problem for Weak Shock Reflection

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This talk focuses on two–dimensional wave structures which arise in modeling weak Mach reflection by 2-D Burgers equation. In spite of the extensive experimental and numerical studies in this field, not very much is known on a theoretical level. Open questions concern the internal structure of two–dimensional elementary waves, bifurcation criteria for two dimensional waves, and the existence theory. We shall present an existence result for a free-boundary value problem leading to a solution which resembles weak Mach reflection with an embedded rarefaction wave. The methods are based on monotone operator techniques and Schauder's fixed point theorem.

On the Use of Multidimensional Characteristic Theory for Solving the 2D Steady Euler Equations in Conservation Form [a]

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Presented by J.-C. Carette

In two space dimensions, the unsteady equations form a hyperbolic system of conservation laws which admits an infinite number of characteristic surfaces $S(x, y, t) = 0$. such that particular linear combinations of the governing equations called compatibility equations involved only derivatives along those surfaces. The space operator in these compatibility equations involves a space-like derivative in the space plane, and a time-like derivative along a time-space path.
This contribution will describe a class of methods which allows a conservative shock-capturing discretization of the Euler equations and which , at the same time, consists of a monotone upwind discretization of the time-like derivatives for a set of well selected compatibility equations.
For supersonic flow, the choice is made such that the space-like direction coincides with the time-like direction, projected on to the space plane, thus effectively leading to an upwinding along the characteristics of steady supersonic flow. For subsonic flow, an algebraic continuation is made, involving upwinding along 4 acoustic directions which cover the omnidirectional domain of dependence and become mutually perpendicular at Mach number $M = 0$.
The monotone advection schemes used are based on shock-capturing SUPG Finite Elements or, alternatively, on positive high resolution multidimensional advection schemes, both for linear triangles. As an illustration, the flowfield in the inlet of a scram jet is shown in Figure 2, with sharp and monotonic capturing of discontinuities.

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[b]Ph.D candidate supported by a Belgian government IRSIA fellowship

Shock Capturing and Global Existence of Entropy Solutions

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In this talk we discuss recent efforts in developing shock capturing methods and corresponding compactness frameworks to solve analytically global entropy solutions to systems of conservation laws, especially the compressible Euler equations in several space variables. Such shock capturing schemes are analyzed to construct efficient approximate solutions. Corresponding existence theorems for global entropy solutions with large initial data are established by developing compensated compactness ideas. Then we show how this approach is successfully applied to solving the compressible flow (Euler and Euler-Poisson) with geometrical structure including transonic nozzle flow, spherically symmetric flow, and symmetric rotating flow.

Conservation Laws for the Relativistic Fluid Dynamics

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The equations of the relativistic fluid dynamics are

$$\begin{aligned} \frac{\partial}{\partial t}\left(n(1-v^2)^{-1/2}\right) + \frac{\partial}{\partial x}\left(nv(1-v^2)^{-1/2}\right) &= 0 \\ \frac{\partial}{\partial t}\left((\rho+p)\frac{v}{1-v^2}\right) + \frac{\partial}{\partial x}\left((\rho+p)\frac{v^2}{1-v^2}+p\right) &= 0 \\ \frac{\partial}{\partial t}\left((\rho+p)\frac{v^2}{1-v^2}+\rho\right) + \frac{\partial}{\partial x}\left((\rho+p)\frac{v}{1-v^2}\right) &= 0 \end{aligned} \tag{1}$$

Here n is the rest mass density, ρ is the proper energy density, p is the pressure and v is the particle speed.
In this talk, we will develop the mathematical theory of the relativistic fluid dynamics. Although the relativistic system is much more complicated and the results are much harder to obtain, we proved that all the parallel results in the classical theory hold for the relativistic hydrodynamics.We also showed that the Newtonian limits of our results are indeed what we have obtained for classical systems.
We obtain a simple formula of the speed of sound in the relativistic background

$$\sqrt{p_\rho} \tag{2}$$

This is much simpler than Taub's original formula which is given by

$$\sqrt{\frac{n}{1+i}\left(\frac{di}{dn}\right)_s} \tag{3}$$

where $i = (e+p)/n$ is the rest specific enthalpy. We proved that the Lax shock inequalities are satisfied globally and entropy is monotone along the shock curves. Because of the complexity of the relativistic system, we developed some new techniques and criterion. By using the Riemann invariants, we introduced a simple criterion to exclude the formation of vacuum. This criterion can be used in general hyperbolic systems to detect the loss of strict hyperbolicity. Since Courant and Friedrichs, people begin to use $(p-v)$ plane to solve the Riemann problem. But a seemingly paradoxical situation, whereby it appears that a Riemann problem admits two distinct admissible solutions, restricts its applications. In this paper, we showed that the $p-v$ plane can be used to uniquely determine all the states in solving the Riemann problem, as long as we keep in mind of the correct entropy level. Then we prove the existence and uniqueness of the solution of the Riemann problem.

A Model Numerical Scheme for Phase Transitions in Solids

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Presented by Bernardo Cockburn

We devise a simple finite difference scheme that produces approximations to the viscosity-capillarity solutions of the equations that govern the propagation of phase transitions in solids (or to the equations of van der Waals fluids) for all positive values of the adimensional parameter that characterizes the viscosity-capillarity solution. Numerical experiments showing the convergence of the method and that no spurious oscillations or spikes are present in the approximate strains are presented.

A Finite Difference Scheme, Fourth-Order in Time and Space, for the Linearized Elastodynamics Equations

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Presented by Gary Cohen

The purpose of our study is to construct a minimal fourth order finite difference scheme to solve the 2D linearized elastodynamics system written in terms of displacement variables in a non-homogeneous medium. This system consists in two coupled second order hyperbolic equations in which the differential operators in space can be expressed by using gradient and divergence operators. The approximation of the derivatives is constructed for the second order differential operators and provides stencils confined in five points in both directions. The approximation in time cannot be obtained by fourth order finite difference for such schemes are unconditionally unstable. For this reason, we use a modified equation approach which consists in adding a fourth order differential operator in space constructed from the square of the global matrix of differential operators in space to the equation semi-discretized in time by a second order centered finite difference scheme. This correcting term is then approximated as the square of a second order centered approximation of the global operator. The presence of the square of the time-step multiplying the correcting term ensures a global fourth order accuracy in time and space.
The stability of the resulting scheme is then studied and we show that the scheme is stable in the homogeneous case and conditionally stable in the non-homogeneous case.
Absorbing and non-absorbing boundary conditions are introduced by coupling with a second order scheme. Modelizing a free surface boundary condition by non-homogeneous zones provides a gain of accuracy.
Finally, a numerical study of the method is done.

High-Resolution Numerical Methods for Low-Mach Number Flows

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In this talk, we will discuss various aspects of extending a collection of algorithmic techniques developed for time-dependent hyperbolic problems to the case of low-Mach number, advectively-dominated fluid flows. These methods include higher-order Godunov methods with limiters, local mesh refinement, and volume-of-fluid representations of fronts and solid surfaces. Primary design issues include finding the appropriate splitting of the equations into hyperbolic and elliptic/parabolic terms, and the careful exploitation of the appropriate forms of locality (respectively, finite propagation speed and local elliptic regularity) for each. We will present examples from incompressible and nearly incompressible flow, combustion, and flows in complicated geometries.

Magnetohydrodynamic Calculations Using a Second Order Godunov Scheme with Adaptive Mesh Refinement

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Presented by Leo P. O'Hare

We present a second-order Godunov, adaptive mesh refinement method for the solution of the one- and two-dimensional equations of ideal magnetohydrodynamics (MHD). In our Godunov scheme we have tried two approaches to approximating the fluxes on the zone boundaries: the first is a simplified flux approximation based on the work of S. F. Davis (SIAM J. Sci. Statis. Comput., 1988); the second is a modified EO flux approximation as in Zachary, Malagoli, and Colella (SIAM J. Sci. Comput., 1994). These schemes have been incorporated into an adaptive mesh code (developed at Lawrence Livermore National Laboratory by John Bell, et al.) which implements the technique of Berger and Colella (JCP, 1989). Brio and Wu (JCP, 1988) showed that the ideal MHD equations are not genuinely nonlinear and so admit compound wave solutions. Using a coplanar MHD Riemann problem, we demonstrate that the Godunov scheme using the simple flux approximation will capture these compound waves. We compare our one-dimensional results with those presented by Brio and Wu, and Zachary, et al. A two-dimensional explosion simulation using both the Davis flux and the modified EO flux is compared to the results of Zachary, et al. Multidimensional results will demonstrate the power and efficiency of using adaptive mesh refinement.

[a]Work supported by the NSWC Independent Research Office.

Riemann Problems for Nonlinear Maxwell's System

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Presented by Yves Combes

We present the adaptation of several numerical schemes of type FDTD (*FFinite Difference in Time-Domain*) for CFD (*Computational Fluid Dynamics*) for the resolution of some problems in *quasilinear monodimensional propagation issuing* of Maxwell's system. The nonlinear constitutive equations used, *permit us both to find an exact Riemann solver* and to study actual cases of nonlinear materials well known [illegible] si-cists. Many configurations of media are considered
There is a growing interest in nonlinear effects in e[illegible] netic phenomena. Theoretical studies have been [illegible] years, [8] and [1]). More recently FDTD methods ha[illegible]en used to solve the full Maxwell's equations and to compute nonlinear effects, [3] and [12]. Furthermore nonlinear effects have been studied theoretically and numerically in fluid dynamics. [2] and [5], and performant CFD methods have been developed [4] and [6]. The aim of this presentation is to show how to adapt some of these CFD methods to nonlinear electromagnetism.
We consider the *Plane waves case* of the Maxwell's system in free space. It can be written:

$$\left\{ \begin{array}{lllll} \partial_t \left\{ \begin{array}{c} D_x \\ B_y \end{array} \right\} & + & \partial_z \left\{ \begin{array}{c} H_y \\ E_x \end{array} \right\} & = 0 & (S_1) \\ \partial_t \left\{ \begin{array}{c} D_y \\ B_x \end{array} \right\} & - & \partial_z \left\{ \begin{array}{c} H_x \\ E_y \end{array} \right\} & = 0 & (S_2) \end{array} \right. \tag{1}$$

The quasilinear constitutive equations studied, permit us to regard *both systems (S_1) and (S_2) independently*. For example, we focus our attention on the first (S_1), which has the form of a system of conservation laws :

$$(T) \left\{ \begin{array}{l} \partial_t U + \partial_x F(U) = 0 \text{ in } \Omega\times]0,T[\\ U(x,0) = U_0(x) \text{ given.} \end{array} \right. \tag{2}$$

where $F(\mathbf{U}) = (H,E)^T$ involves an electromagnetic energy density $W(D,B)$ such as $E = \frac{\partial W}{\partial D}$ and $H = \frac{\partial W}{\partial B}$.
The convexity of W corresponds to the hyperbolicity of (T), and provides us an entropy criterion for unicity [9]:

$$\partial_t W + \operatorname{div} \left(\frac{\partial W}{\partial \mathbf{D}} \wedge \frac{\partial W}{\partial \mathbf{B}} \right) \leq 0. \tag{3}$$

We have introduced special constitutive relations permitting us to find both Riemann invariants when the fields are genuinely nonlinear, according to the work of [10], and then to solve the Riemann problem for (T).
An exact Riemann solver has been found – the solution is constituted of constant states separated by either shock waves (or contact discontinuities) or rarefaction waves. Then we have implemented some numerical second order TVD schemes based on approximate Riemann solvers [11].
For the problem in a single medium, the Roe average [7] is unique and has been found simply.
In the case of two media, the continuity of fluxes involves the Maxwell's transmission conditions at the interface. We can found also an exact Riemann solver, but now the Roe solver isn't unique.
Those schemes are finally applied to the computation of the propagation of an electromagnetic wave through a Kerr medium. A comparison is made with results given by the frequential approach and with FDTD computations.

[a]Contrat de Formation-Recherche CEA/DAM.

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Quasineutral Asymptotic for the Euler-Poisson System

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Presented by Stéphane Cordier

A fluid 1D model of plasma consisting of electrons and ions is considered. The electrons are described by their density n_e, their velocity u_e and their temperature T_e; the ions by n_i, u_i and T_i respectively. These state variables satisfy the scaled equations of conservation of mass, momentum and energy:

$$\begin{aligned}
\partial_t n_\alpha + \partial_x(n_\alpha u_\alpha) &= 0,\\
\partial_t(n_\alpha m_\alpha u_\alpha) + \partial_x(n_\alpha m_\alpha u_\alpha^2 + n_\alpha T_\alpha) &= n_\alpha q_\alpha E,\\
\partial_t W_\alpha + \partial_x((W_\alpha + n_\alpha T_\alpha)u_\alpha) &= n_\alpha q_\alpha u_\alpha E,
\end{aligned}$$

with $\alpha = e$ for the electrons and i for the ions, $q_i = -q_e = 1$ their charge, $m_i = 1$ and $m_e = \eta \ll 1$ their mass and $W_\alpha = \frac{1}{2} n_\alpha m_\alpha u_\alpha^2 + \frac{3}{2} n_\alpha T_\alpha$ their energy. These equations are coupled by the scaled Poisson equation

$$\lambda^2 \partial_x E = n_i - n_e,$$

where the small parameter λ is the scaled Debye length. When $\lambda \to 0$, the Euler-Poisson system leads formally to a nonlinear hyperbolic system in a nonconservative form called the quasineutral Euler system for which the jump relations are not determined. The admissible shock solutions of the quasineutral Euler system are defined as the weak limits of travelling wave solutions of the Euler-Poisson system when $\lambda \to 0$. The travelling wave analysis is based on a phase plane analysis of an equivalent dynamical system and leads to three generic type of travelling wave solutions:

- Solitary wave solutions
- Shock wave solutions
- Periodic shock solutions

depending on the Mach number value. These different shock profiles can be related to the collisionless shock structures described by the plasma physicists. Only one of the three types of solutions tends to non trivial weak solutions of the quasineutral system when $\lambda \to 0$. Then, the Riemann problem for the quasineutral Euler system can be solved with the obtained jump relations.

A High-order Godunov Scheme for Ideal Magnetohydrodynamical Equations Based on an Nonlinear Riemann Solver

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Presented by Wenlong Dai

A high-order Godunov scheme is developed for multidimensional ideal magnetohydrodynamical (MHD) equations on the base of an nonlinear Riemann solver and the operator splitting method. Its correctness and robustness are demonstrated through numerical examples involving strong shocks.
A Riemann problem in ideal MHD equations may involve seven waves, each of which may be discontinuous. The Riemann solver used in the scheme is constructed from conservation laws across any discontinuity and assumes all discontinuities present in a Riemann problem. For a given set of left and right states, the solver first guesses four values for the magnetic field and its orientation. Then the solver does the calculations for two possible fast shocks, for two possible rotational discontinuities, and for two possible slow shocks, and obtains two post-shock states for the slow shocks. Finally the solver uses the condition for a contact discontinuity to improve the initial guess, and then go back to the first step. The solver, if iterated to convergence, requires a few iterations, but for the approximate calculation for time-averaged fluxes needed in the scheme, one or two iterations is found to be sufficient.
Our multidimensional scheme is based on the operator splitting method. Each pass (e.g., x-pass) of the scheme updates all the variables except B_x which is updated in a y-pass. The scheme introduces neither any source term due to the dimensional projection nor any additional step to formally insure the divergence-free condition. The conservation of magnetic fluxes across shocks in our multidimensional scheme is demonstrated through numerical examples. The nonstrict hyperbolicity of the equations is also addressed and our approach to treat the resulting singularity is presented.

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Gas and Oil Combustion in Porous Media

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Presented by Jesus Carlos da Mota

We discuss theoretical aspects of oil and gas displacement in a porous medium under a combustion process. One typical application of this theory is "in-situ-combustion", a method used for heavy oil recovery. One question is: To which extent is the flow determined by the initial states, provided there is one sharp reaction front moving into the combustible fluid through the porous medium?
We present a simplified non linear system of partial differential equations which model the phenomena behavior. Compressibility and volumetric changes associated with combustion process are neglected. However, heat conduction, chemical reaction rates and capillar pressures present in multiphase fluid displacement in porous media are taken into account. We discuss the traveling waves solutions joining burned to non burned states for this simplified system. Studying the orbits of the associated dynamic system, we determine all initial states for which a reaction front is possible. We draw a parallel with the classical theory of Chapman-Jouguet for combustion in gases and identify in our model the different regimes corresponding to deflagrations and detonations in that theory.

A Short Course on Parallel Computing

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This course is designed for both academic and industrial scientists interested in parallel computing and its applications to large-scale scientific and engineering problems. We will explain the application of a class of distributed-memory MIMD supercomputers (Paragon, CM-5, and T3D) to such real-world problems as molecular dynamics-aided design and manufacturing, electromagnetic scattering, and protein folding.
We will cover the parallel computing basics including hardware, software, and algorithm design, in addition to programming these machines for some simple problems such as domain decomposition for PDEs, linear algebra (solving $Ax = B$ and computing eigenpairs), global optimization (Monte Carlo methods), as well as molecular dynamics. These problems are representative of a large portion of current applied science issues. This short course emphasizes practice and understanding of basic concepts of parallel computing.

Systems of Conservation Laws with a Singular Characteristic Field

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We analyze the behavior of a linearly characteristic field of a generic non-strictly hyperbolic system of three conservation laws. The system describes three phase flow in porous media and can model polymer injection or thermal processes to improve oil recovery. Under the hypothesis of the Implicit-Function Theorem we obtain the existence of a curve consisting of points where the line field is singular. For the case corresponding to the polymer injection, we obtain the curve of singular points explicitly and we show that the line field has a saddle behavior. The curve of singular points coincides with the secondary bifurcation locus of Hugoniot branches associated to contact discontinuities with crossing structure.

The Efficiency of Numerical Methods for Solving Multivariate Scalar Conservation Laws

Ronald A. DeVore

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This talk will discuss the potential efficiency of numerical methods for solving multivariate scalar conservation laws such as Finite Element Methods, moving grids, and wavelets. The potential efficiency of such methods is connected with the regularity of the solution to the conservation law. Thus part of this talk will discuss what we know about the regularity of solutions. We will point out the need for new methods to measure regularity.

On the Stability of Viscous Profiles for Scalar Conservation Laws with Discontinuous Flux Function

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Presented by Stefan Diehl

Consider the scalar conservation law with discontinuous flux function

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(H(x)f(u) + \left(1 - H(x)\right)g(u)\right) = 0, \qquad (1)$$

where H is Heaviside's step function. This type of equation arises in continuous sedimentation of solid particles in a liquid, in two-phase flow and in traffic-flow analysis. There are no convexity conditions on f or g. Solutions of this equation possess a non-unique discontinuity at $x = 0$. The equation can be written as a 2×2 triangular non-strictly hyperbolic system and, depending on f and g, the discontinuity at $x = 0$ is either a regular Lax, under- or over-compressive, marginally under- or over-compressive or a degenerate shock wave. By a viscous profile we mean a stationary solution of $u_t + (F^\delta(u,x))_x = \epsilon u_{xx}$, where F^δ is a smooth approximation of the discontinuous flux, i.e., H is smoothed. We present some results on the stability of these profiles, i.e., the decay to zero of small disturbances as $t \to \infty$. This is done by weighted energy methods, where the different weights (depending on f, g and the profile) play a crucial role. In terms of the 2×2 system, there is only a disturbance in the first equation. If the total initial mass of the disturbance is zero, we show stability for any type of wave. For non-zero initial mass we can handle some cases, which will be presented in the talk.

Theory of Conservation Laws in China

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In this lecture, a report of the history and main contributions on conservation laws in China will be given systematically.

On Second Order "Completely Exceptional" Hyperbolic Conservation Equations which are Linearizable

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Presented by Andrea Donato

Nonlinear wave propagation problems are described by quasi-linear or nonlinear hyperbolic equations whose solution usually breaks down in a finite time. If one considers, for instance, a single equation of order n, the jump in the higher order derivatives of the field u propagate along the characteristics with speeds depending on the field u and its derivatives until the order $n-1$ and determined as roots of the characteristic polynomial. Of course, also the characteristic velocities have, in general, discontinuities in the first order derivatives across the characteristics. If one looks to the law of propagation of discontinuities, then one realizes that such discontinuities in the speeds of propagation are responsible for the occurrence of the breakdown in the $(n-1)$-order derivatives of the solution. In some sense this occurrence is typical of the nonlinearity.
However, it may happen that a speed of propagation depends on the field u and its derivatives in such a way that its first order derivatives are not discontinuous. In this case the corresponding wave is said to be "exceptional". If all the speeds of propagation have this property the equation is called "completely exceptional" and behaves like a linear equation in the sense that the breakdown in the $(n-1)$-order derivatives of the solution will not occur. A typical example of a nonlinear equation having this property is the Monge-Ampère equation supposed to be hyperbolic.
On the other hand, in a recent paper it has been shown that, under suitable conditions, there exists at least in one, two and three spatial dimensions, a special type of Bäcklund transformation of reciprocal type which maps the Monge-Ampère equation to a linear canonical form. Then, in some sense, there is a link between the conditions of complete exceptionality and the linearization of nonlinear equations, at least under special circumstances. This type of approach is valid also in more than one space dimension and allows to characterize classes of equations which can be linearized.
In this paper we give a procedure leading to the linearization of second order conservative hyperbolic equations by requiring the conditions of "complete exceptionality"; the procedure involves not only second order equations in conservative form but also nonhomogeneous first order systems. More precisely, we shall consider the (1+1)-dimensional second order conservative equation

$$\partial_t u_t + \partial_x F(u, u_t, u_x) = 0, \qquad (1)$$

and the (2+1)-dimensional second order conservative equation

$$\partial_t u_t + \partial_x F(u, u_t, u_x, u_y) + \partial_y F(u, u_t, u_x, u_y) = 0; \qquad (2)$$

then, by assuming the given equations to be hyperbolic, we shall require the condition of complete exceptionality and find the conditions allowing their reduction to linear form.

Numerical Solutions of Conservation Laws Using Central Difference Schemes

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We analyze numerical solutions of conservation laws, where the solution contains both shocks and contact discontinuities. For problems in 1-D we find that these two kinds of discontinuities should be treated differently.
In the shock layer region a first order artificial viscosity term shall be switched on. The amount of artificial viscosity is determined by the eigenvalue that changes sign in the shock. A fast moving shock can be calculated in a grid that moves with approximately the speed of the shock, to avoid smearing.
The region around the contact discontinuity, as well as the rest of the solution, can be calculated with a second order accurate scheme. The scheme has been constructed by considering the phase error which is introduced by a standard central difference scheme. We extend these ideas to 2-D.
Numerical solutions of the 1-D Riemann problem where the two techniques are implemented will be demonstrated. Also, numerical results of a model problem for contact discontinuities in 2-D will be presented.

2D Wave Interactions for Materials with Nonconvex EOS: Computational Results

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Presented by E. Erskine

2D computational results for 3 × 3 gas dynamics with shock wave interactions in materials with nonconvex EOS are presented. A BCT variant of the second order Godunov scheme is used in the calculations.
The numerical method, as it applies to these problems, is outlined. Our code is validated by comparison with prior shock-on-wedge results (using a convex only code) and associated experiments. New 2D shock-on-wedge wave interactions are presented for both "fabricated" and real EOS. 1D studies are also presented. Finally, the older 2 × 2 code has been exercised for an EOS case with a cusp and these results are also included.

Boundary Conditions for Hyperbolic Problems

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Presented by Bernardo Favini

The paper illustrates the application of a compact matricial formulation [1] to the numerical solution of compressible inviscid flows. The present paper is confined to the analysis of smooth flows. The proposed method adopts the wave front model [2,3], which can be considered as an extension of the λ scheme approach to non-orthogonal grids. According to the wave front model, the component-wise form of the governing equations written in a non-orthogonal curvilinear system of coordinates can be cast into a compact matrix form as

$$\boldsymbol{w}_{,\tau} + \sum_{i=1}^{N_d} R_i \Lambda_i L_i T \hat{\boldsymbol{w}}_{,i} = 0 \qquad (1)$$

In the equations above, the symbol $\boldsymbol{w}$ denotes the algebraic vector of the dependent variables where the velocity vector $\boldsymbol{q}$ is projected over the co-variant base, whereas the notation $\hat{\boldsymbol{w}}$ is used when $\boldsymbol{q}$ is projected over the Cartesian base. The spatial co-variant derivatives of $\boldsymbol{w}$ are evaluated by means of standard partial derivatives of $\hat{\boldsymbol{w}}$ [5] according to the rule

$$\boldsymbol{w}|_i = T\hat{\boldsymbol{w}}_{,i} \qquad (2)$$

where T is a transformation matrix defined as a function of the curvilinear mapping jacobian. The numerical integration of system (1) is obtained by means of an explicit two-step characteristic-biased scheme [3,4,9], which is second-order accurate both in time and space. At each step, it uses a two point stencil for each wave direction, and for three-dimensional problems is stable for a CFL number less than 2/3. Relations (2) allow to express the co-variant derivatives without the need of introducing the undifferentiated Christoffel symbols, whose characteristic-biased discretization is computationally expensive. A TVD modification presented in [10] makes the scheme oscillation free.
The adoption of the wave front model and of the co-variant base to represent vectors and differential operators allows a simple and accurate enforcement of the boundary conditions. When system (1) is evaluated at a boundary point, some of the quantities

$$\mathcal{L}_j^k = \lambda^k \boldsymbol{l}^k T \hat{\boldsymbol{w}}_{,j} \qquad (3)$$

will correspond to waves carrying information inwards the computational domain. Each of these missing pieces of information has to be replaced by relations modelling the outside world in order to obtain a well-posed problem at that boundary point [5,7]. Following [2,5,6], these boundary conditions can be formulated in differential form. Thus, a condition expressing the time invariance of some integral quantity can be written as a, generally non-linear, combinations of the time derivative of $\boldsymbol{w}$

$$\boldsymbol{b} \cdot \boldsymbol{w}_{,\tau} = B \qquad (4)$$

or, similarly, a relation on spatial gradients can be cast as a combinations of the partial derivatives of

$$\boldsymbol{s} \cdot \hat{\boldsymbol{w}}_{,j} = S \qquad (5)$$

These relations must be rewritten as conditions upon $\mathcal{L}_j^k$. In the full paper, the analysis of the integration at the boundaries will be addressed in detail. The compact formalism will ease the discussion of the compatibility along edges and at corners of different types and combinations of boundary conditions. This part of the work completes the analysis presented in [8].

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Multidimensional High Order Schemes for Systems of Conservation Laws

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For the class of scalar conservation laws, the theory of convergence and stability for a large number of numerical methods is very well established. This is also true in view of error estimates and convergence properties, i.e. the order of convergence. Even for multidimensional calculations there are more and more attempts to design high order schemes.

For the class of systems of conservation laws, the picture differs from the previous one. For most of the existing schemes, there are only hand-waving arguments, that these methods are of the same order as their scalar counterparts. If we look at numerical methods for multidimensional systems it gets even worse. Here, Strang showed that most of the methods are limited to at most second order accuracy.

Based on a truly multidimensional numerical scheme, called method of transport, that we proposed in the last conference on Hyperbolic Problems in 1992, we shall present a more general class of multidimensional schemes. In a theorem we will summarize the necessary conditions on a set of waves such that the resulting method is first-order consistent with the compressible Euler equations. With the help of this theorem, we can show that the Euler equations can be decomposed in a finite number of advection equations with variable coefficients in any space dimensions. Some numerical examples show the robustness and simplicity of the resulting method.

A rigorous error analysis of this method shows that independently of the accuracy of the numerical scheme to solve the advection equation, the overall scheme is at most first-order accurate. The same analysis shows the solution to this problem. There exist correction terms to the advection equations, such that the sum of the corrected equations approximates the non-linear system with the same order of accuracy as the simple advection equations are. In addition to this, the order of accuracy is not limited to two, as in the dimensional splitting case, and it is independent of the space dimensions.

Structure and Stability of Radiative Shock Waves

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Presented by Doris Folini

We present results of the structure and stability properties of radiative shock waves and the numerical techniques we use. The problems arising from a stiff source term are discussed.

As radiative shock waves are a common feature in astrophysical objects, a proper understanding of their structure and stability is essential for modeling astrophysical flows. We investigate the time evolution of radiative colliding flows, using the inhomogeneous Euler equations. A source term in the energy equation describes the radiative cooling of the post-shock flow in a parameterized form as a function of temperature and density.

We numerically find the interaction zone of the two colliding flows to contain a previously unobserved hot layer (see Figure 3a) which is thermally unstable. The cold dense layer which establishes in this region due to radiative cooling is found to be dynamically unstable (see Figure 3b).

From a numerical point of view, the simulations are demanding in several ways. As a spatial resolution of at least six orders of magnitude is necessary, we use the adaptive mesh refinement algorithm of Berger [1] (see also Figure 3a). To include the source term we use a Strang splitting. This works fine as long as the source term is not stiff. A stiff source term, however, can lead to wrong wave speeds. From an astrophysical point of view the results are important as the newly found hot region can contribute considerably to the X-ray emission of the entire interaction zone. The instabilities may contribute to the complex structures of astrophysical nebulae as well as to their observed varyability. Finally, from a mathematical point of view, the results give new insights into the stability properties of solutions to the special kind of inhomogeneous Euler equations we use.

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Nonlinear Waves in a Turbulent Compressible Two-Equation Model

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Presented by A. Forestier

We examine herein the behaviour of the first order differential part of the standard k-ϵ compressible model (with special attention paid to the associated Riemann problem), making use of the Favre averaging process; this system writes (see [6] for instance):

$$(\rho)_{,t} + (\rho U_i)_{,i} = 0$$

$$(\rho U_i)_{,t} + (\rho U_i U_j)_{,j} + P_{,i} + \frac{2}{3}K_{,i} = 0$$

$$(E)_{,t} + (U_j(E + P + \frac{2}{3}K))_{,j} = 0$$

$$(K)_{,t} + (U_j K)_{,j} + \frac{2}{3}(U_j)_{,j} = 0$$

$$(\epsilon)_{,t} + (U_j \epsilon)_{,j} + \frac{2}{3}C_{\epsilon_1}\epsilon(U_j)_{,j} = 0$$

with an associated averaged perfect gas state law, i.e. : $P = (\gamma - 1)(E - \frac{1}{2}\rho U_j U_j - K)$. We focus first on the case where the specific heat ratio is smaller than 5/3. The latter system, which does not possess a conservative form, is a non-strictly unconditionally hyperbolic system which is invariant under frame rotation. Its eigenvalues write:

$$\lambda_1 = U - c' \quad ; \quad \lambda_{2 \to 6} = U \quad ; \quad \lambda_7 = U + c'$$

where c' denotes some modified sound celerity, i.e. : $c'^2 = \gamma\frac{P}{\rho} + \frac{10}{9}\frac{K}{\rho}$. Moreover, the last equation weakly couples with the remaining. It is shown here that the one dimensional Riemann problem associated to system (1) (and given left (L) and right (R) states) does admit a unique solution provided that:

$$U_R - U_L < \frac{2}{\gamma - 1}(Z_R + Z_L) \quad \text{with} \quad c = (\gamma\frac{P}{\rho})^{1/2} \leq Z \leq c'$$

which degenerates to the standard laminar gas dynamics case (see [5]). Moreover, this unique solution does fulfill the realisability requirement (which means that the mean density, mean pressure and the turbulent kinetic energy remain positive through shocks, contact discontinuities and rarefaction waves). This requires to introduce approximate jump conditions, using theoretical results provided in [1], [2] and [4]. The parametrization of Riemann invariants, contact discontinuities and shocks is then given. The final solution is obtained solving a nonlinear system with two unknowns, unlike in the gas dynamics case (see [5]). The latter requires the development of a specific solver.

Some computations performed in a one dimensional framework are given, which provide the main wave patterns of the turbulent kinetic energy, choosing a standard specific heat ratio ($\gamma = 1.4$). The influence of the path connecting two different states through shocks will be eventually discussed, together with the influence of the constant value of C_{ϵ_1}. The whole model is such that viscous terms are consistent with the entropy concept. The counterpart of the present study when γ is chosen to be greater than 5/3, which is detailed in [3], will be shortly presented.

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On the Stability of Non-Classical Shock Waves

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For a system

$$\mathbf{u}_t + (f(\mathbf{u}))_x = \epsilon(B(\mathbf{u})\mathbf{u}_x)_x, \quad \mathbf{u} : \mathbf{R} \times [0, \infty) \to U \subset \mathbf{R}^n,$$

of conservation laws, consider shock waves

$$\mathbf{u}(x,t) = \begin{cases} u^-, & x < st \\ u^+, & x > st \end{cases} \qquad (\text{for } \epsilon = 0)$$

and

$$\mathbf{u}(x,t) = \phi((x - st)/\epsilon) \text{ with } \lim_{\xi \to \pm\infty} \phi(\xi) = u^{\pm} \text{ (for } \epsilon > 0),$$

and let

$$R^{\pm}(u,s) \equiv \sum_{\pm(\lambda - s) > 0} \ker (Df(u) - \lambda I), \quad (u,s) \in U \times \mathbf{R}.$$

Classical Laxian shock waves satisfy

$$R^-(u^-, s) \oplus R^+(u^+, s) \oplus \mathbf{R}(u^+ - u^-) = \mathbf{R}^n \qquad (1)$$

and are known, under suitable assumptions, to be stable in appropriately defined senses.
This talk presents various results on the stability of shock waves which violate (1). Some of the results seem to contribute to the issue of admissibility criteria.

A Hysteretic Polymer Flooding Model

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It is well-known that multi-phase flow in porous media exhibits hysteresis; this is typically modeled by modifying the saturation dependence of the relative permeabilities. The solution for the Riemann problem for polymer flooding shows cases with non-monotonic behavior in saturation. This suggests hysteresis could be important.
In this talk we describe the global solution of the Riemann problem for 3-component 2-phases polymer flooding with hysteresis. We show that hysteresis produces more complicated solutions. We also discuss the questions of non-uniqueness of solutions.

Functional Solutions and Approximate Methods Convergence

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We present the new approach to background of approximate methods convergence based on functional solutions theory [1] for conservation laws. The applications to physical kinetics, gas and fluid dynamics are considered .

Let W be a locally compact metric space with a Borel measure μ, dx and dt are the Lebesgue measures on $\mathbf{R}_n$ and $\mathbf{R}_1$ respectively; $\dot{B}$ is the set of bounded compactly supported Borel functions on the topological product $Q = W \times \mathbf{R}_n \times \mathbf{R}_1$; $Q_0 = W \times \mathbf{R}_n; \dot{B}^\infty$ are the functions infinitely differentiable with respect to (x,t), whose derivatives belong to $\dot{B}$. Suppose $\mathcal{D} \subset L_1^{loc}(W,\nu), f_j : \mathcal{D} \times \mathbf{R}_n \times \mathbf{R}_1^+ \longrightarrow L_1^{loc}(Q,\nu), 1 \le j \le n$, are defined mappings, where $\nu = \mu \otimes dx \otimes dt$. On the set of locally summable functions $M = \{u\}$ we consider the system of equations for the unknown variable $u \in M$

$$\int_Q [u\partial_t g \quad + \quad \sum_{j=1}^{n}(f_j \circ u)\partial_{x_j} g + (f_{n+1} \circ u)]\nu(dQ)$$
$$+ \quad \int_{Q_0} g|_{t=0} u_0 \nu_0(dQ_0) = 0, \forall g \in \dot{B}^\infty,$$

which is used as a definition of a generalized solution of the Cauchy problem for a system of conservation laws with initial data u_0. The extension of the concept of a solution proposed in this paper (functional solutions [1]) makes it possible to justify the convergence of approximate methods in the presence of an *a priori* estimate of an approximation in $L_1^{loc}(Q,\nu)$, which is uniform in the parameter.

Theorem 1 *Suppose the method $\mathcal{M}$ is stable and there is property of weak approximation. Then this method converges to global regular functional solution of Cauchy problem (1).*

Theorem 2 *Every regular functional solution is defined by unique element of space $L_1^{loc}(Q,\nu)$.*

Theorem 3 *Suppose the compact set K consists of regular functional solutions which are produced by compact set Y of initial data and there exists the unique element in K for every initial data in Y. Then functional solutions in K are continuously dependent of initial data in Y if K and Y are equipped by topology of weak convergence in L_1^{loc} .*

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Viscosity Approximating Solutions to ODE Systems that Admit Shocks, and Their Limits

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We study nonlinear systems of ordinary differential equations that arise when considering stationary one dimensional systems of conservation laws with forcing terms defined in a bounded interval. We construct weak entropy solutions of bounded variation which are pointwise and $\mathcal{L}^1$ limits of solutions of regularized, i.e., viscous, systems, where the limit is taken in the viscosity parameter. In particular, no oscillations occur either for the viscous solutions or for the inviscid one. We also discuss the possible formation of boundary layers when boundary values are prescribed for the viscous regularized equations. As applications, first we show the existence of transonic solutions of bounded variation with strong shocks for the equation of stationary gas flow in a duct of variable area as a pointwise limit of artificial viscosity solutions. We analyze their properties depending on the kind of duct as well as on the boundary data of the regularized problem. Second we show the model applies to the hydrodynamic modeling for semiconductor devices for some particular heat conduction terms and added diffusion to the energy equation. In particular, we show that under the assumption of bounds for the state variables, there exists a regular solution for that particular viscous heat conducting model. Also, if the bounds for the state variables are uniform in the vanishing parameters, we get existence of an inviscid weak entropy solution of bounded variation as a pointwise limit of the regular ones.

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Numerical Simulations in Two Dimensional Granular Flow

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We present some results on an antiplane shear problem for two dimensional granular flow.
In this problem, the equations describing the motion lose hyperbolicity under loading and become ill-posed. Shear Bands are introduced as strong discontinuities that localize the plastic deformation at the point of ill-posedness. We present an algorithm for the numerical study of these problems. This algorithm is based on a higher order Godunov method. Near the shear band, we use front tracking techniques that allow for unlimited growth of the band at the tips (there is no a priori bound for this rate of growth) and unloading relief along the band.

Quantum Hydrodynamic Simulation of Hysteresis in the Resonant Tunneling Diode

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The quantum hydrodynamic (QHD) model [1] approximates quantum effects in the flow of electrons in a semiconductor device by adding quantum corrections to the classical electrogasdynamical equations. The leading $O(\hbar^2)$ quantum corrections have been remarkably successful in simulating the effects of electron tunneling through potential barriers including single and multiple regions of negative differential resistance in the current-voltage curves of resonant tunneling diodes (the archetypal quantum device).
In this talk, I will present QHD simulations of hysteresis in a double barrier resonant tunneling diode at 77 K [2]. These are the first simulations of the QHD equations to show hysteresis. The simulations demonstrate that bistability is an intrinsic property of the resonant tunneling diode, in agreement with experimental observations and with simulations of the Wigner-Boltzmann equation.
Hysteresis appears in many settings in fluid dynamics. The simulations presented here show that hysteresis is manifested in the extension of classical fluid dynamics to quantum fluid dynamics.
For the simulations, a finite element method for the time-dependent QHD model is introduced [2]. The finite element method is based on a Runge-Kutta discontinuous Galerkin method for the QHD conservation laws and a mixed finite element method for Poisson's equation and the source terms in the QHD conservation laws.

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A Model of a Plug-Chain System Near the Thermodynamic Critical Point: Connection with the Korteweg Theory of Capillarity and Modulation Equations

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Presented by Sergey L. Gavrilyuk

We consider the plug flow of a gas-liquid mixture in a pipe of constant area. We assume that the liquid and gas are different and non reacting chemicals and we consider the case when the gas is near the thermodynamic critical point. The appropriate nonlinear models of the flow pattern are derived: a discrete plug-chain model and its long wave approximation. The explicit solutions of both models are obtained.
The mechanical analogy in the discrete case is derived. The mechanical analogy represents an infinite chain of mass points which move only in the transversal direction.
It is found that the model for the long-wave approximation of a plug-chain system can be considered as a special example of the Korteweg theory of capillarity.
The modulation equations of the Korteweg theory of capillarity are obtained. The averaging quantities are described by Godunov's system of quasilinear partial differential equations. Examples of hyperbolic systems of modulation equations are given.

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Solving the Compressible Euler Equations in Time-Dependent Complex Geometries

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I shall present a numerical algorithm to solve the non-stationary three-dimensional compressible Euler equations in complex time-dependent geometries. This algorithm is an upwind finite volume method working on a grid of tetrahedra. Such kinds of grids allow great flexibility in approximating complex three-dimensional geometries. Furthermore, local adaption of the grid to (non-stationary) flow phenomena (shocks, steep gradients, vortices and singularities) can be used without additional effort in the numerical scheme. Another possibility to achieve high resolution of flow phenomena is to use a numerical scheme of higher order in space. I shall present a new cell-centered finite volume scheme of second order in space working on a grid of simplices. This approach is motivated by a theoretical result of convergence in the case of scalar conservation laws in two space dimensions. Tests with this approach show that this really is of higher order for problems with smooth solutions and the used limiter function avoids oscillations at discontinuities.
For the local adaption of the grid, a criterion is necessary which controls the fineness of the grid according to the numerical error of the scheme. For the system of the Euler equations, no a-posteriori error estimator is available as for elliptic or parabolic problems. Therefore, many heuristic criteria, for instance oriented on large gradients, differences or second derivatives, are used. An approach based on the residuum is well motivated for scalar conservation laws in 1D with a result of Tadmor. Therefore an interpretation of the residuum approach for the system of the Euler equation will be discussed.
The combination of solving conservation laws and moving boundaries of the geometry demands a numerical scheme which guarantees conservation. A new approach to fulfill that for a finite volume scheme working on simplices will be presented.
The presented algorithm has been applied to calculate the flow in a complex cylindrical geometry with a moving piston (see Fig. 4).

[a]This work has been supported by the Deutsche Forschungsgemeinschaft.

The Two-Dimensional Riemann Problem for the Linearized Gas Dynamic Equations

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We give the explicit solution of the two-dimensional Riemann problem for the linearized gas dynamic equations on a structured or unstructured mesh. We first derive a d'Alembert equation for the pressure P. Then we use Kirchhoff formulae to formally define P and, as a consequence, the density ρ and the two components of the velocity. Finally the explicit formulae obtained are proven to define the unique solution of the bidimensional Riemann problem.

Study of "Residual" Boundary Conditions

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Presented by Marguerite Gisclon

We consider a hyperbolic system of conservation laws perturbed by a viscous term :

$$(**)\begin{cases} \frac{\partial}{\partial t}(u^\epsilon(x,t)) + \frac{\partial}{\partial x}(f(u^\epsilon(x,t))) \\ \qquad = \epsilon\frac{\partial}{\partial x}(B(u^\epsilon(x,t))\frac{\partial}{\partial x}u^\epsilon(x,t)),\ x>0,\ t>0, \\ u^\epsilon(x,0) = b(x),\ x>0, \\ u^\epsilon(0,t) = a(t),\ t>0, \end{cases}$$

where $u^\epsilon(x,t) \in \mathbf{R}^n$, $f : \mathbf{R}^n \to \mathbf{R}^n$ smooth, $B \in GL_n(\mathbf{R})$. We suppose that, for all u, the eigenvalues of $Df(u)$ are real and satisfy :

$$\lambda_1(u) < \dots < \lambda_p(u) < 0 < \lambda_{p+1}(u) < \dots < \lambda_n(u).$$

We denote by u the solution of the hyperbolic system with "residual" boundary conditions :

$$(*)\begin{cases} \frac{\partial}{\partial t}u(x,t) + \frac{\partial}{\partial x}f(u(x,t)) = 0,\ x>0,\ t>0, \\ u(x,0) = b(x),\ x>0, \\ u(0,t) \in C(a(t)),\ t>0, \end{cases}$$

where

$$C(a) = \{\bar{u} \in \mathbf{R}^n,\ \exists\, v(y) \in \mathbf{R}^n \text{ solution of } (P)\},$$

and the problem (P) is defined by

$$(P)\begin{cases} B(v(y)+\bar{u})v'(y) = f(v(y)+\bar{u}) - f(\bar{u}), \\ v(0)+\bar{u} = a, \\ v(+\infty) = 0. \end{cases}$$

We suppose that
1) the hyperbolic system $(*)$ admits an entropy E such that $D^2E = S$ is a symmetric positive definite matrix.
2) the viscosity matrix B satisfies

$$\exists\, \alpha > 0, \forall\, u \in \mathbf{R}^n, \forall\, \xi \in \mathbf{R}^n,\ (S(u)\xi, B(u)\xi) \geq \alpha|\xi|^2,$$

then we prove:

Theorem 1 *The set $C(a)$ is a manifold in a neighborhood of the point a with dimension p whose tangent space is $E_a^s = \bigoplus_{i=1}^p \ker(Df(a) - \lambda_i(a)I_n)$.*

Theorem 2 *There exists a time $T > 0$, such that u^ϵ converges towards u in $L^\infty(0,T,L^2(\mathbf{R}^+))$ as ϵ tends to zero.*

A Well Balanced Scheme for the Numerical Processing of Source Terms in Hyperbolic Equations

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In a variety of physical problems one encounters source terms which are balanced by internal forces and this balance supports multiple steady state solutions which are stable. Typical of these are gravity-driven flows such as those described by the shallow water equations over a nonuniform ocean bottem (see (1.10)). Many classical numerical scheme cannot maintain these steady solutions or achieve them do not preserve the proper balance between the source terms and internal forces. We proposed here a numerical scheme, adapted to a scalar conservation law, which preserves this balance and which can, hopefully, be extended to more general hyperbolic systems. The proof of convergence of this scheme towards the entropy solution is given and some numerical tests are reported.

Global Solutions of the Relativistic Euler Equations in Non-Flat Spacetimes

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The existence of solutions with shocks is demonstrated for the equations describing a perfect fluid in general relativity on an $n \geq 2$ dimensional spacetime, namely, $divT = 0$, where $T^{ij} = (p + \rho c^2)u^i u^j + pg^{ij}$ is the stress-energy tensor for a perfect fluid, and the divergence is the covariant divergence. Here p denotes the pressure, u the n-velocity, ρ the mass-energy density of the fluid, g^{ij} the metric of the manifold, and c the speed of light. The special equation of state $p = \sigma^2 \rho$ is assumed, where σ^2, the sound speed, is assumed to be constant. The metric is restricted to the class of conformally flat metrics, a class which includes the Robertson-Walker metric, a metric used to model an expanding universe. For these equations we construct bounded weak solutions of the initial value problem which travel in one of the space dimensions for any initial data of finite total variation. A non-flat metric appears in these equations as a source term and, in general, to solve equations with a source term by Glimm's method in general requires that the source term, as well as the total variation of the initial data, be small. However, in the extreme relativistic limit, all equations of state tend to $p = \frac{c^2}{n-1}\rho$, and for this special case we can assume arbitrarily large initial total variation and show that the total variation of solutions remains uniformly bounded in time, independent of the strength of the gravitational field.

Gaussian-Based Moment-Method Closures for the Solution of the Boltzmann Equation

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Presented by Clinton P. T. Groth

This paper is concerned with the development of higher-order moment closures and generalized transport equations that provide approximate solutions to the Boltzmann equation of gaskinetic theory. The goal is to provide improved mathematical modeling for the numerical solution of rarefied flows in the transitional regime, beyond but near the hydrodynamic (continuum) limit, and specific applications would include problems in both the physics of space plasmas and upper-atmospheric aerodynamics of hypersonic vehicles. Desirable features of closure methods are positivity of the approximate particle velocity distribution functions and hyperbolicity of the associated transport equations for they ensure finite speeds of propagation and well-posedness of initial boundary value problems. Previous closure models based on perturbations to a Maxwellian cannot be easily endowed with these properties. New higher-order moment closures are proposed based on Chapman-Enskog like expansions for the particle distribution function in terms of a novel non-isotropic Gaussian velocity distribution. The expansion technique permits the inclusion of the hydrodynamic stresses in a non-perturbative fashion and effectively extends the methodology's range of validity by avoiding the breakdown of the transport equations. The relevant mathematical theory and desirable properties of the non-isotropic Gaussian distribution and moment closures are described. The proposed quasi-linear symmetric transport equations are shown to be stable and hyperbolic, and the associated particle phase-space distribution functions are demonstrated to be positive, for a significant range of physical conditions (i.e., well defined regions of phase space in the vicinity of the Gaussian limit), making them both tractable and well-suited for modern numerical solution algorithms.

A Numerical Study of Shock Interactions and Shock Induced Mixing

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Presented by John W. Grove

This talk will describe several applications of the front tracking method to simulations of shock accelerated interfaces. Three main applications will be discussed, the acceleration of fluid interfaces in a shock tube, the refraction of an expanding shock wave through a thermal boundary layer at a wall, and the acceleration of closed fluid interfaces by expanding shocks. The important issues here are the early structure of the shock refractions, and the later time chaotic mixing at the fluid interfaces. We will also discuss the benefits of tracking the two dimensional wave interactions produced by the shock refractions by comparing simulations where these interactions are tracked with corresponding simulations where they are captured.

Fourth Order Difference Methods for Hyperbolic IBVP

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We consider fourth order difference approximations of initial-boundary value problems for hyperbolic partial differential equations. We use the method of lines approach with both explicit and compact implicit difference operators in space. The explicit operator satisfies a summation by parts condition leading to an energy estimate and strict stability.
Compact implicit difference operators are based on an approximation $\frac{\partial}{\partial x} \to P^{-1}Q$, where P and Q are non-diagonal difference operators In this way the error constant can be substantially reduced, and the extra work required for solving the band systems in each time step may well pay off. No general theory is currently available for this type of approximations. We present a complete stability analysis for the implicit fourth order approximation by generalizing the Laplace transform technique. We construct boundary conditions such that the resulting approximation is strongly stable and gives a fourth order global error estimate.
We also present numerical experiments for the linear advection equation and the Burger's equation with discontinuities in the solution or in its derivative. The first equation is used for modeling contact discontinuities in fluid dynamics, the second one for modeling shocks and rarefaction waves. The time discretization is done with a third order Runge-Kutta TVD method. For solutions with discontinuities in the solution itself we add a filter based on second order viscosity.
For the non-linear Burger's equation we use a flux splitting technique that results in an energy estimate. This splitting also guarantees that the entropy condition is fulfilled. In particular we shall demonstrate that the unsplit conservative form produces a non-physical shock instead of the physically correct rarefaction wave.
In the numerical experiments we compare our fourth order methods with standard second order ones and with TVD-methods of order one, two and three. The results show that the *fourth order methods are the only ones that give good results* for all the considered test problems.

The Cauchy Problem for an Elastic String with a Linear Hooke's Law

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Presented by Harald Hanche-Olsen

The equations of planar motion of a string take the form

$$u_t - \left(\frac{T\xi}{r}\right)_x = 0 \tag{1}$$

$$v_t - \left(\frac{T\eta}{r}\right)_x = 0 \tag{2}$$

$$\xi_t - u_x = 0 \tag{3}$$

$$\eta_t - v_x = 0 \tag{4}$$

after scaling, where the string is parametrized by its length x at rest, $\xi = X_x$, $\eta = Y_x$, $u = X_t$, and $v = Y_t$. Here $(X(x,t), Y(x,t))$ is the position of the point labeled x on the string at time t. We assume the (scaled) tension T is given by the linear Hooke's law

$$T = r - 1 \text{ where } r = \sqrt{\xi^2 + \eta^2} \tag{5}$$

and $r = 1$ corresponds to the absence of any tension. For a rigorous derivation of these equations, see Antman [1].
Keyfitz and Kranzer [3] considered the string with a *nonlinear* force law. In that setting, the transversal waves made up a linearly degenerate family, while the longitudinal waves formed a nondegenerate family.
For a linear law, both these families become linearly degenerate, and so both families support contact discontinuities. In addition to the contact discontinuities, we find certain anomalous shocks corresponding to a 180° bend in the string, but these shocks either create or destroy energy. For this reason, and because the Riemann problem has a unique solution without any anomalous shocks, we dismiss the anomalous shocks as unphysical.
Energy conservation leads to L^2 estimates for the solution. Thus we are able to show that Glimm's scheme [2] produces a weak solution to the Cauchy problem for arbitrarily large initial data by working in L^2 spaces.

[1] Stuart S. Antman, "The Equations for Large Vibrations of Strings," *Amer. Math. Monthly*, vol. 87, pp. 359–370, 1980.

[2] James Glimm, "Solutions in the Large for Nonlinear Hyperbolic Systems of Equations," *Comm. Pure Appl. Math.*, vol. 28, pp. 697–715, 1965.

[3] Barbara L. Keyfitz and Herbert C. Kranzer, "A System of Non-Strictly Hyperbolic Conservation Laws Arising in Elasticity Theory," *Arch. Rat. Mech. Anal.*, vol. 72, pp. 219–241, 1980.

Adaptive Multiresolution Schemes for Shock Computations

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In this talk we present adaptive multiresolution schemes for the computation of discontinuous solutions of hyperbolic conservation laws. Starting with the given grid, we consider the grid-averages of the numerical solution for a hierarchy of nested grids which is obtained by diadic coarsening, and compute its equivalent multiresoluton representation. This representation of the numerical solution consists of the grid-averages of the coarsest grid and the set of errors in predicting the grid-averages of each level of resolution from the next coarser one; these errors depend on the size of the grid and the local regularity of the solution. At a jump-discontinuity they remain of the size of the jump , independent of the level of resolution; this observation enables us to identify the location of discontinuities in the numerical solution. In a region of smoothness, once the numerical solution is resolved to our satisfaction at a certain locality of some grid, then the prediction errors there for this grid and all finer ones are small; this enables us to obtain data compression by setting to zero terms of the multiresolution representation that fall below a specified tolerance. The numerical flux of the adaptive scheme is taken to be that of a standard centered scheme, unless it corresponds to an identified discontinuity, in which case it is taken to be the flux of an ENO scheme. We use the data compression of the numerical solution in order to reduce the number of numerical flux evaluations as follows: We start with the computation of the exact numerical fluxes at the few grid-points of the coarsest grid, and then proceed through diadic refinement to the given grid. At each step of refinement we add values for the numerical flux at the new grid-points which are the centers of the coarser cells. Wherever the solution is locally well-resolved (i.e. the corresponding prediction error is below the specified tolerance) the costly exact value of the numerical flux function is replaced by an accurate enough approximate value which is obtained by an inexpensive interpolation from the values of the coarser grid.

Global Solutions of the Navier-Stokes Equations for Multidimensional Compressible Flow with Discontinuous Initial Data

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We prove the global existence of weak solutions of the Navier-Stokes equations for compressible, isothermal flow in two and three space dimensions when the initial density is close to a constant in L^2 and L^∞, and the initial velocity is small in L^2 and bounded in L^{2^n} (in two dimensions the L^2 norms must be weighted slightly). A great deal of qualitative information about the solution is obtained. For example, we show that the velocity and vorticity are relatively smooth in positive time, as is the "effective viscous flux" F, which is the divergence of the velocity minus a certain multiple of the pressure. We find that F plays a crucial role in the entire analysis, particularly in closing the required energy estimates, understanding rates of regularization near the initial layer, and most important, obtaining time-independent pointwise bounds for the density.

Nonlinear and Compressible Effects in the Richtmyer-Meshkov Instability

Richard L. Holmes and John W. Grove
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David H. Sharp
Los Alamos National Laboratory

Presented by Richard Holmes

The Richtmyer-Meshkov instability is generated when a shock wave strikes an interface between two gases causing perturbations on the interface to grow with time. There has been a great deal of interest in this instability due to its importance in Inertial Confinement Fusion and supernova theory.
So far theories of the Richtmyer-Meshkov instability which predict the growth rate of interface perturbations have failed to agree with the results of shock tube experiments. These theories include the impulsive model of Richtmyer and the linearized theory of Richtmyer and its recent generalization by Yang, Zhang and Sharp. We present the the results of numerical simulations which indicate that the small amplitude and incompressibility assumptions made by these theories are not valid over the timescales considered in experiments and that this invalidity is likely to be the reason for the lack of agreement.

A Level Set Formulation for Interfacial Fluid Flows

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A level set formulation is derived for incompressible, immiscible Navier-Stokes equations separated by a free surface. The free surface is identified as the zero level set of a smooth function. The effects of discontinuous density, discontinuous viscosity and the surface tension can be taken into account naturally. High order front capturing finite difference methods are proposed based on this level set formulation. These methods are robust, efficient, and are capable of computing topological transition and interface singularities such as merging and reconnection of fluid bubbles. Numerical experiments of bubble merging and roll-up of a jet are presented to demonstrate the effectiveness of the methods.

Nonlinear Diffusive Phenomena of Solutions for the System of Compressible Adiabatic Flow

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Consider the system

$$\begin{cases} v_t - u_x & = & 0 \\ u_t + p(v,s)_x & = & -\alpha u, \ \alpha > 0 \\ s_t & = & 0 \end{cases} \tag{1}$$

which can be used to model the adiabatic gas flow through porous media. where v is specific volume, u denotes velocity, s stands for entropy, p denotes pressure with $p_v < 0$ for $v > 0$. It can be proved that the solutions of (1) tend to those of the following nonlinear parabolic equation time-asymptotically.

$$\begin{cases} v_t & = & -\frac{1}{\alpha}p(v,s)_{xx} \\ s_t & = & 0 \\ u & = & -\frac{1}{\alpha}p(v,s)_x \end{cases} \tag{2}$$

[a]Research supported in part by National Natural Science Foundation of China.

Slow Dynamics of Linear Waves in Nonlinear Systems of Conservation Laws

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Presented by F. Hubert

Hunter and Russo posed the problem of the description for large time of perturbation of hyperbolic systems of conservation laws:

$$\begin{cases} u_t^\epsilon + div(f(u^\epsilon)) = \epsilon P[u^\epsilon], \ u^\epsilon \in R^n, \ x \in R, \ t > 0, \\ u^\epsilon(x,0) = a(x), \ x \in R. \end{cases}$$

Such systems appear in several models of continuum mechanics, the perturbation $\epsilon P[u]$ (usually a PDO) might so represent the effect of viscosity, heat conduction etc... We notice that ϵ is a small parameter, $x \mapsto a(x)$ a periodic function and we suppose that the system for $\epsilon = 0$ is hyperbolic and endowed by both linear and nonlinear characteristic fields We note $\lambda(u)$ the spectral value of $Df(u)$ corresponding to the linear field, $\tau = \epsilon t$ the slow time, $c = c(\tau)$ the solution of $\frac{d}{d\tau}(\tau c(\tau)) = \lambda(u)$. Observing that for time $t \ll \epsilon^{-1}$ and smooth initial data, the perturbation $\epsilon P[u]$ can often be ignored , so that the nonlinear waves will be damped, D. Serre gave a qualitative description of the dynamics for times $t \sim \epsilon^{-1}$. If we use a change of variable which isolates the non linear waves ($v^\epsilon \in R^{n-1}$) from the linear ones ($w^\epsilon \in R$), we can consider the asymptotic development:

$$\begin{cases} v^\epsilon(x,t) = v_0(\tau) + \epsilon v_1(x - c(\tau)t, \tau) + \mathcal{O}(\epsilon^2), \\ w^\epsilon(x,t) = w_0(x - c(\tau)t, \tau) + \epsilon w_1(x - c(\tau)t, \tau) + \mathcal{O}(\epsilon^2). \end{cases}$$

In first approximation the nonlinear modes depend only of the slow time, whereas the linear mode is a wave whose speed is also a function of the slow time.
The aim of this talk is to show various results obtained in that way, for the system of Keyfitz and Kranzer and for the system of gas dynamics. We can prove for these two systems, that the homogenized problem (system satisfied by v_0 and w_0) is well posed.

Optimal Grid of the Steady Euler Equations

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Presented by W. H. Hui

The Euler equations for steady supersonic flow may be solved using a shock-capturing methodology by marching in a spatial coordinate. When the Eulerian formulation is used with cartesian coordinates (x, y), the computation suffers from two drawbacks: (a) sliplines are smeared badly and (b) the marching (in x, say) fails due to ill-posedness in regions where the x-component of velocity is subsonic, although the total velocity there is supersonic. The generalized Lagrangian formulation of Hui and his coworkers [1], in which the coordinates are (λ, ξ) with ξ being a stream function, eliminates the first drawback while partially remedies the second.

In this paper, we have found an optimal coordinate system, and thus the optimal computational grid, in the generalized Lagrangian formulation for which the coordinate lines λ = const and ξ = const are orthogonal. In comparisons using test problems, we find that computing the Euler equations based on the optimal grid has the following advantages: (a) it is most robust, being well-posed everywhere, (b) it has fewer equations to solve and, (c) it gives more accurate results. In addition, orthogonality of the optimal coordinates ensures no large distortion of the computational cells in the physical plane, thus eliminating the need of conventional Lagrangian formulation to re-map to the Eulerian plane and its resulting errors.

Figure 5 shows the flow-generated grid and the computed Mach number in a channel flow. The computation was done using the adaptive first order Godunov scheme, and the exact analytical solution was reproduced numerically. The computation terminates at B where the Mach numbers is 1.000719. When Eulerian formulation or other non-orthogonal streamlined coordinates were used, the computation could not proceed beyond the point A due to ill-posedness of the local Cauchy Problem.

[1] W. H. Hui, *J. Comput. Physics*, vols. 89, 207, 103, 450, and 465.

A Completely Integrable Hyperbolic Variational Equation

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Presented by John K. Hunter

We show that the following nonlinear partial differential equation,

$$(u_t + uu_x)_{xx} = \frac{1}{2}\left(u_x^2\right)_x,$$

is a completely integrable, bi-Hamiltonian system. The corresponding equation for $w = u_{xx}$ belongs to the Harry-Dym hierarchy. This equation is the canonical asymptotic equation for weakly nonlinear solutions of a class of hyperbolic equations derived from variational principles. It is also the high-frequency limit of the integrable Camassa-Holm equation which is a model equation for shallow water waves. Using the bi-Hamiltonian structure, we derive a recursion operator, a Lax pair, and an infinite family commuting Hamiltonian flows, together with the associated conservation laws. In addition we find the transformation to action-angle coordinates

Smooth solutions of the partial differential equation break down in finite time because their derivative blows up. Nevertheless, we show that there is a class of piecewise linear energy-conserving weak solutions, for which the Hamiltonian structure and complete integrability remain valid even after their derivative blows up. We compute explicitly the bi-Hamiltonian structure on the finite dimensional invariant submanifolds of piecewise linear solutions which is obtained by restricting the bi-Hamiltonian structure of the partial differential equation

Modelling Relativistic Flows with Modern High-Resolution Shock-Capturing Schemes

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Presented by José M$^{\underline{a}}$. Ibáñez

We have extended *modern high-resolution shock-capturing schemes* to the multidimensional relativistic hydrodynamics system of equations. In order to carry out this extension we *have analysed the spectral decomposition of the Jacobian matrices* associated to the fluxes. The interest of this analysis, both from the theoretical and from the numerical point of view, is discussed. Two severe tests show the performance of our numerical hydro-code. Several astrophysical applications are displayed for which the correct modelling of formation and propagation of strong shocks is of crucial importance.

A Mixed Type Equation for Dipole Chains

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Presented by E. Infeld

A simple dipole chain is the main building block of many models in solid state physics. Here we consider nearest neighbour interactions of the individual charges of the dipole. In the dipole approximation, a mixed type nonlinear differential equation, more general than sine-Gordon, governs the dipole angles. Soliton solutions are found. Collisions of these solitons are investigated numerically. These collisions do not seem to be elastic. On the other hand, theoretical arguments seem to point at the equation being integrable.
Extensions of the equation to include finite dipole moments, etc., have been found.

Flux Calculation for Multiphase Flow in Porous Media

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We consider the system of equations arising from multiphase flow in porous media when capillary effects are neglected. Besides a small region, this system is usually hyperbolic. To simulate numerically such flows using cell-centered finite differences, engineers have designed an appropriate flux calculation that is called the upstream mobility flux.

This calculation is formulated implicitly in the sense that to calculate the flux, which is the flow rates of the phases, one needs to know the directions to which the phases are flowing, and this is part of what we are calculating. We show that by ordering the phases according to their densities one can make this calculation explicit.

In the case of two-phase flow where the system reduces to a scalar conservation law, we show that the upstream mobility flux is an approximate Riemann solver with all the desired properties for convergence of the associated cell-centered finite difference method and we compare it to more standard numerical fluxes.

Finally we consider the case where the porous medium is discontinuous at a discretization point. Then the flux function of the conservation law is discontinuous with respect to space at that point. We show how to calculate a numerical flux based on Godunov's flux at the interface between rock types and we compare it numerically to the upstream upstream mobility flux.

A Kinetic Formulation for Chromatography

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Presented by François James

We present a new example of kinetic formulation for a hyperbolic system of conservation laws, which is a simplified model for chromatography. The kinetic formulation consists here in writing a set equations for a whole family of entropies. Namely, for a system of N equations, we obtain $N+1$ kinetic representations, from which we recover the invariant regions and the stability for weakly convergent initial data by a compensated compactness argument, including the non-strictly hyperbolic case.

On the Courant-Friedrichs-Lewy Condition Equipped with Order for Hyperbolic Differential Equations

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Presented by R. Jeltsch

In 1928 Courant, Friedrichs and Lewy showed a necessary condition for convergence of a difference scheme for solving a hyperbolic differential equation. This condition states that the numerical domain of dependence has to include the exact domain of dependence. Applying this argument to the linear advection equation, where the exact domain of dependence is given by the characteristic line, says that an explicit difference stencil must have at least one stencil point on both sides of the characteristic line trough the stencil point on the newest time level. In 1985 this result has been generalized to explicit and implicit, normalized, two time level difference schemes in the form that a stable scheme with local error order p must have at least $\lceil \frac{p}{2} \rceil$ stencil points on each side of the characteristic line. It is conjectured that this statement is correct for multistep difference schemes. In the present paper we consider in particular three time level schemes and can show the correctness of the conjecture in certain cases while in others we give strong evidence which support the conjecture.

A Cell Entropy Inequality for Discontinuous Galerkin Method in General Triangulations

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Presented by Guangshan Jiang

In this talk we will discuss a cell entropy inequality for high order discontinuous Galerkin method in general triangulations for the square entropy. The inequality holds with or without slope limiters. Since similar result holds for finite difference schemes (those evolving one piece of information per cell) only under much more restricted cases (one dimension, convex, second order, strong slope limiter), this result shows a theoretical advantage and potential of discontinuous Galerkin method or similar methods evolving not just one piece but several pieces of information per cell.

Implicit Numerical Schemes for Hyperbolic Conservation Laws with Stiff Relaxation Terms

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We are concerned with the underresolved numerical schemes for hyperbolic conservation laws with stiff relaxation terms. Spurious numerical results, some of them have not been reported elsewhere, are observed and investigated through the correct asymptotic limit analysis. We find that these unphysical results are caused by inappropriate temporal discretizations, thus can be eliminated with improved temporal integrators. A second order splitting method is developed, which has the correct asymptotic limit even if the initial layer and the small relaxation time are not numerically resolved.

Numerical Integrations of Systems of Conservation Laws of Mixed Type

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The systems of conservation laws have been used to model dynamical phase transitions in, for example, the van der Waals fluid and the propagating phase boundaries in solids. When integrating such mixed hyperbolic-elliptic systems the Lax-Friedrichs scheme is known to give the correct solutions selected by a viscosity-capillarity criterion *except* a spike at the phase boundary which does not go away even with a refined mesh. We identify the source of this spike as an inconsistency between the Lax-Friedrichs discretization and the viscosity-capillarity equations, and show a simple change of variable that can eliminate this spike. We then implement the relaxation schemes for the mixed type problems that select the same viscosity-capillarity solutions as the Lax-Friedrichs scheme with higher resolutions. Furthermore, a flexibility in the relaxation schemes is used to obtain solutions for a wide range of the viscosity-capillarity equations.

Convergence of Approximate Solutions to 2×2 Systems of Hyperbolic Conservation Laws

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We present a compactness framework theorem for the convergence of approximate solutions to general 2×2 systems of nonstrictly hyperbolic conservation laws. These systems arise in applications in multiphase flows in porous media, magnetohydrodynamics, and elasticity. We apply this framework theorem to a canonical class of quadratic 2×2 systems that are nonstrictly hyperbolic at isolated points in the state space. We prove the convergence of approximate solutions generated by the vanishing viscosity method, the Godunov scheme, and the Lax Friedrichs scheme. As a direct consequence, we obtain the existence of global weak entropy solutions to these systems with arbitrarily large initial data in L^{∞}. Our proof uses compensated compactness and involves a very detailed and complicated analysis of the wave curve geometry and the singularities of solutions to a highly singular generalized Euler-Poisson-Darboux type equation.
We will also discuss some preliminary results on the corresponding initial boundary value problem for these nonstrictly hyperbolic systems as well as for general strictly hyperbolic systems. In particular, we are interested in certain nonlinear boundary conditions and their relationship with boundary layers and the well-posedness of the problem.

Shock Refraction at a Cylindrical Bubble Interface

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Presented by Smadar Karni

The interaction of shock waves with bubble inhomogeneities is often studied as a model to elucidate the mechanism of vorticity/turbulence generation and mixing enhancement. The passage of the shock through the bubble occurs over a short time-scale during which the dynamics is dominated by complex shock refraction, diffraction and reflection at the bubble interface. The bubble is compressed and accelerated and its shape is substantially deformed. Shear is generated at the bubble interface due to misalignment between the incident shock front and the bubble interface. The interface becomes unstable, it continues to evolve slowly and rolls-up to generate a pair of vortex lines or a vortex ring.
A numerical method that we have developed is used to simulate shock/bubble interactions. The multi (in this case two) component flow is modelled by the compressible Euler equations augmented by a species evolution equations. The fluid components are assumed ideal gases in mechanical equilibrium, occupying the same space with complementary concentrations. The pressure is computed using the effective ratio of specific heat of the mixture. The main features of the method are: (i) a primitive (nonconservative) discretization scheme, which automatically maintains pressure equilibrium among the gas components, a notorious difficulty with conservative discretizations. Viscous correction terms control conservation errors; (ii) a parallel adaptive mesh refinement implementation, an absolute necessity as brute force computations are not a viable option for this type of simulations. (A parallel computation of our simulations that took two evenings to complete on a cluster of 8 workstation would need over 900 hours to run on a single processor of a CRAY Y-MP, without adaption); and (iii) schlieren-type flow visualization images, instrumental in elucidating the subtleties of the phenomena involved.
We used our scheme to reproduce two experiments of a shock wave in air interacting with a light (Helium) and a heavy (Refrigerant 22) cylindrical bubbles, reported by Haas and Sturtevant (*J. Fluid Mech.*, 1987). The speed of sound in Helium/R22 is higher/lower than in air, thus the two setups yield very different evolutionary patterns. The experiments/simulations concentrate on the early part of the interaction, which is dominated by repeated refractions and reflections of acoustic fronts at the bubble interface. Detailed comparison of computational results with experimental shadowgraph images shows *remarkable* qualitative and quantitative (within 4%-5% in velocity measurements) agreement. The simulations faithfully reproduce the intricate geometry of the wavefronts and volume deformations, and may serve to explain the complicated interactions (transition from regular to irregular refraction, cusp formation and focusing, folding processes, multi-shock and Mach shock structures, jet formation etc.), and possibly shed light on other subtle flow features that are either hard to measure or less well understood.

[a] Supported in part by an NSF Postdoctoral Fellowship, NSF grant #DMS92 03768, ONR grant #N00014-92-J-1245 and a Packard Fellowship to Leslie Greengard

Special Element Flux-Corrected Method for High-Speed Compressible Viscous Flows

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Presented by George Em Karniadakis

A hybrid spectral element/Flux-Corrected Transport method was used successfully in [1] to circumvent the problem of Gibbs phenomena in the numerical simulation of problems with discontinuities using spectral methods ([2]). Here we report on its extension to the two-dimensional Navior-Stokes equations. The equations are cast in conservation form and solved in time via a fractional step procedure, splitting the inviscid from the viscous part. In that frame, the underlying Gibbs phenomena acquire a new importance, in view of the higher-derivatives needed to form the viscous fluxes, in the presence of discontinuities (i.e., shocks). We propose a nevel technique consisting of the use of viscous flux limiting (Flux-Corrected-Diffusion), based on a physical argument along the lines of FCT. The result is a simple, transparent scheme unifying the treatment of both the hyperbolic and the parabolic part of the Navier-Stokes operator under one formulation. The usefulness of the final algorithm is enhanced by its multi-domain nature, permitting consideration of flows in complex geometries. Results are presented for 2-D subsonic and supersonic Flow over a bluff body ([3]), which bring out important physical information. Although the numerical results concern exclusively fluid-dynamical problems, the proposed treatment is applicable to all problems where a hyperbolic and a parabolic operator co-exist and in which physical arguments of conservation apply ([4]).

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[2] D. Gottlieb and S. A. Orszag, *Numerical Analysis of Spectral Methods. Theory and Applications.*, SIAM, Philadelphia, 1977.

[3] A. Roshko, "Perspectives on Bluff Body Aerodynamics," in *Proceedings of the Second International Colloquium on Bluff Body Aerodynamics and Applications*, Melbourne, Australia, December 7-10, 1992.

[4] John G. Giannakouros, Ph.D. thesis, Dept. of Mechanical/Aerospace Engineering, Princeton University, 1994.

A Class of Parametrices for Certain Hyperbolic Operators

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An extension is proposed of the L^p-theory of pseudodifferential operators obtained by integration of a multi-parameter family of oscillating symbols. While the classical continuity of the arising operators in Sobolev- and Lebesgue-type spaces holds, the construction is distinguished from the standard theory by the properties of anisotropic smoothing and of the controlled expansion of a singular support of a function (distribution) under the action of these operators.

The corresponding symbolic calculus includes an appropriate asymptotic expansion formula, a correctly defined symbolic composition, as well as the invariance under linear transformations. These properties are applied to the invertibility problem for a certain class of hyperbolic operators as illustrated in the two dimensional case by the following.

Theorem. Let $P : S'(R^N) \to S'(R^N)$, $N = 2$, be a hyperbolic operator

$$(Pu)(x,y) \equiv u_{xx} - u_{yy} + a_1 u_x + a_2 u_y + a_0 u,$$

with real-valued coefficients $a_j(x,y) \in C^\infty(R^N) \cap S'(R^N)$ satisfying

$$\inf_{x,y} (a_1(x,y) \pm a_2(x,y))/(1 + |(x,y)|)^r > 0$$

for some $r < 1$. Then:

(i) $u \in L^p_{loc}(R^N)$ whenever $Pu \in L^p_{comp}(R^N)$;

(ii) $u \in C^\infty(R^N)$ whenever $Pu \in C^\infty(R^N)$;

(iii) P is invertible modulo $OPS^{-\infty}(R^N)$.

The upper value for r is final in the sense that if the condition $r < 1$ were replaced by $r < 1 + \epsilon$ with $\epsilon > 0$ then none of the three conclusions above would have been true.

This construction of parametrix is an extension of ideas in [1] that leads to the solution of an open problem in [2, §2.2] quoted in the title of [1].

[1] N.M.Kasumov, "Calderon-Zygmund Theory for Kernels With Non-Point Sets of Singularities," *Matematich. Sbornik*, vol. 183, pp. 89–106, 1992.

[2] E.M. Stein, *Singular Integrals and Differentiability Properties of Functions*, Princeton Univ. Press. 1970.

Self-Similar Solutions of Multidimensional Conservation Laws

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Presented by Barbara Lee Keyfitz

When Riemann problems for conservation law systems in two space dimensions are written in self-similar coordinates, the resulting problem takes the form of a boundary-value problem for a conservation law system which changes type. This system has some novel features, but some reduced one-dimensional Riemann problems arise which can be solved. Furthermore, in some cases we have been able to solve the full mixed-type boundary-value problem. The solutions obtained have some interesting singularities. Potential applications to quasisteady shock reflection by a wedge will be discussed.

Semi-Implicit High Resolution Schemes for Low Mach Number Flows

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Presented by R. Klein

Modern high resolution shock capturing methods allow the accurate and robust numerical simulation of fully compressible fluid flow. Unfortunately, these schemes become inefficient and inaccurate in the low Mach number regime for two reasons. First, due to the Courant-Friedrichs-Levy time step restriction the number of time steps needed to simulate a nontrivial flow evolution increases as $1/M$, where $M \ll 1$ is a characteristic flow Mach number. Secondly, there is a stringent dynamic range problem, because pressure fluctuations of order $O(M)$ and $O(M^2)$ must accurately be resolved, if the propagation of weak acoustic waves as well as the limiting incompressibility constraint are to be represented appropriately.

A single time, multiple space scale asymptotic analysis yields detailed insight into the solution structure for the Euler equations as the Mach number vanishes. The analysis, in particular, suggests a new two-term flux decomposition of the Euler equations. The first of the resulting sub-systems represents the advection of mass and momentum, while the second system simultaneously describes a possible global compression in confined flows, long-wave acoustic wave propagation and the divergence constraint in the zero Mach number limit.

Motivated by this analysis, we suggest in this paper a general semi-implicit extension of modern high resolution schemes for the low Mach number regime. By using an operator splitting technique for the above mentioned sub-systems, we obtain a class of flow solvers that incorporate the capabilities of both the underlying shock capturing methods for fully compressible flows and a semi-implicit algorithm for low Mach numbers with a purely convection dominated time step.

An important aspect of this approach is the introduction of multiple pressure variables according to an asymptotic expansion $p = p^{(0)} + M\, p^{(1)} + M^2\, p^{(2)}$. This decomposition allows us to pass to the limit of zero Mach number, i.e to incompressible flows, without changing the structure of the numerical scheme. In this limit the single scalar implicit equation that is to be solved in the second split step reduces to a standard Poisson type pressure correction equation known from incompressible flow solvers.

We discuss implementations for several modern high resolution schemes and demonstrate the performance of the semi-implicit versions through solutions of various test problems. These include weakly nonlinear acoustic effects at Mach numbers $M = O(10^{-2} - 10^{-1})$ as well as the advection of large amplitude density fluctuations at Mach numbers as small as $M = 10^{-4}$.

Existence of Solutions to Resonant Systems of Conservation Laws

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Presented by Christian Klingenberg

We consider the Cauchy problem for a model of one-dimensional gas flow with variable duct area:

$$\begin{aligned} \rho_t \quad + \quad (\rho u)_x \quad &= \quad -\frac{a'(x)}{a(x)}(\rho u) \\ (\rho u)_t \quad + \quad (\rho u^2 + p(\rho))_x \quad &= \quad -\frac{a'(x)}{a(x)}(\rho u^2). \end{aligned} \tag{1}$$

We also consider the related system:

$$\begin{aligned} (a\rho)_t \quad + \quad (a\rho u)_x \quad &= \quad 0 \\ u_t \quad + \quad \left(\frac{u^2}{2} + \int^{\rho} \frac{p'(s)}{s} ds\right)_x \quad &= \quad 0, \end{aligned} \tag{2}$$

where ρ is density, u velocity, $p = p(\rho)$ is pressure, and a is a function of the space variable x. Systems (1) and (2) are of interest because resonance occurs. T. P. Liu was the first to study the Cauchy problem for these systems by using Glimm's random choice method [2], [3]. Recently in [1], Isaacson and Temple solved the Riemann problem for a general inhomogeneous system. Our interest in studying this resonant problem is motivated by their papers. Using the method of compensated compactness, *global existence* of weak solutions to these systems of two conservations laws *is shown*. This is achieved through constructing entropy-entropy flux pairs of Lax type and using estimates from singular perturbation theory for ODE's.

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[3] T. P. Liu, "Resonance for a Quasilinear Hyperbolic Equation," *J. Math. Phys.*, vol. 28, 1987.

Hyperbolic Conservation Laws with Source Terms: Errors of the Shock Location

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When computing numerical solutions of hyperbolic conservation laws with source terms, one may obtain spurious solutions — solutions which seem to be correct but which are totally unphysical such as shock waves moving with wrong speeds. Therefore it is important to know how errors of the shock location can be estimated [1], [2].

For our theoretical analysis we investigate a scalar Riemann problem. We compute its solution using a splitting method. This means that in each time step the homogeneous conservation law and an ODE (modeling the source term) are solved separately. We show that the local error of the shock location can be considered to consist of two parts

- one part that is introduced by the splitting
- and another that occurs because of smeared shock profiles.

By means of numerical examples we test how far these error-estimates can be used to adapt the step size so that the error of the shock location remains sufficiently small. The numerical examples include one-dimensional systems.

[1] R.J. LeVeque and H.C. Yee, "A Study of Numerical Methods for Hyperbolic Conservation Laws with Stiff Source Terms," *J. Comput. Phys.*, vol. 86, pp. 187–210, 1990.

[2] P. Colella, A. Majda and V. Roytburd, "Theoretical and Numerical Structure for Reacting Shock Waves," *SIAM J. Sci. Stat. Comput.*, vol. 4, pp. 1059–1080, 1986.

Accuracy Optimized Methods for Stiffly Forced Conservation Laws

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An extension of Accuracy Optimized Methods (AOMs) will be presented for stiffly forced conservation laws of the form

$$u_t + f(u)_x = \psi(u) \qquad (1)$$

Numerical methods that treat the convective and forcing terms separately fail when the forcing term becomes stiff; i.e., when $\Delta t\ \psi'$ becomes large where Δt is the time increment in the numerical scheme. One place where this failure shows up is in the transport of shocks at incorrect speeds.
The direct application of AOMs to the stiffly forced conservation law (1) produces differences related to the integrated conservation laws associated with the original conservation law. For example, with $f(u)$ linear (e.g., $f(u) = au$) differences of the form

$$\Delta\psi_j^n = \psi_{j+\frac{1}{2}}^n - \psi_{j-\frac{1}{2}}^n$$

$$\Delta\theta_j^n = \theta_{j+\frac{1}{2}}^n - \theta_{j-\frac{1}{2}}^n$$

$$\vdots$$

appear for the integrated conservation laws

$$\psi_t + a\psi_x = \psi\psi' = \theta$$

$$\theta_t + a\theta_x = \theta\theta' = \gamma$$

$$\vdots$$

The result is that approximate values of the unknowns in the functionally related stiffly forced conservation laws automatically appear. These conservation laws may be treated with AOMs in the same way the original conservation law is treated. This suggests the iterated application of the AOM framework to functionally related conservation law. The AOM framework allows control of the approximated variable in each conservation law through the imposition of constraints in an optimization problem. Thus approximations of the successive variables, u, ψ, $\psi\psi'$, can be controlled by imposing constraints on each of the conservation laws individually. This allows control of the stiffness parameter $\Delta t\psi'$ in the original conservation law. Computational results will be presented to verify the results presented.

Bounds for Threshold Amplitudes in Subcritical Shear Flows

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Presented by Gunilla Kreiss

A general theory which can be used to derive bounds on solutions to the Navier-Stokes equations is presented. The behaviour of the resolvent of the linear operator in the unstable half-plane is used to bound the energy growth of the full nonlinear problem. Plane Couette flow is used as an example. The norm of the resolvent in plane Couette flow in the unstable half-plane is proportional to the square of the Reynolds number (R). This is used to predict the asymptotic behaviour of the threshold amplitude below which all disturbances eventually decay. A lower bound is found to be $R^{-21/4}$. Numerical examples, which give an upper bound on the threshold curve, predict R^{-1}. The discrepancy is discussed in the light of a model problem.

Multidimensional Wave-Propagation Methods

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A class of high-resolution algorithms for multidimensional conservation laws will be described. These methods are based on solving Riemann problems and propagating the resulting waves in a multidimensional manner. This approach has several advantages in terms of its similarity to one-dimensional algorithms, simplicity of implementation, and ease of handling boundary conditions. It also adapts easily to front tracking methods. Algorithms of this form have been developed for advection in an arbitrary incompressible flow, wave equations, and the Euler equations of gas dynamics. Extension from two dimensions to three dimensions is also very easy. Several different applications will be discussed.

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Numerical Computation of 3-D Rayleigh-Taylor Instability in Compressible Fluids Using Scheme with High Resolution for Contact Discontinuities

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Presented by X. L. Li

In this presentation, we will describe our numerical simulation of 3D Rayleigh-Taylor instability using a second order finite difference scheme with artificial compression. This numerical scheme is similar to the TVD scheme but gives higher resolution on contact discontinuities. The numerical solutions are compared to the exact 3D solution in the linear regime, the numerical solution using the TVD scheme and the solution using a front tracking method. The computational program is parallelized for simulation on Intel/iPSC-860 and using PVM on a SUN cluster. We will present our results of 3D simulation on single bubble evolution, bubble merger and chaotic mixing in Rayleigh-Taylor instability.

Conservation Laws for Two-Layer Flows of Mixible Fluids

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The Cauchy problem correctness for nonlinear equations of continuum mechanics is as yet not adequately investigated, especially for systems of mixed type. For some equations describing unsteady flows it is possible that an initially hyperbolic system becomes an elliptic one on a time-dependent solution. To understand physical processes preventing such situations in real flows a mixed-type system may be considered as an equilibrium one for a more complicated model.

For two-layer flows a hierarchy of mathematical models taking into account mixing and generation of short waves at the interfaces is developed. An intermediate mixing layer is taken in the model as the third layer. A three-layer flow scheme is preferable to a two-layer one since mass, momentum and energy are conserved in the flow and the governing equations consist of conservation laws. Therefore, the well-known contradiction connected with the proper choice of conservation laws in the multilayer shallow water theory may be overcome, and internal hydraulic jumps are uniquely determined.

It is shown that the mathematical model containing no empirical constants represents the main peculiarities of entrainment and downstream control in mixing layers and buoyant jets. In particular, the model explains an essential difference in the entrainment rate for subcritical and supercritical flows. This phenomenon is directly related to the fact that equilibrium conditions for a turbulent mixing layer can be fulfilled only for subcritical flows where the equilibrium system is hyperbolic. These models are used also to describe the mixing and blocking effects in two-layer flow over an obstacle. Some applications to geophysical processes are given.

A Two-dimensional Riemann Solver for Hydro-Elastic-Plastic Materials

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Presented by X. Lin

It was shown in [1] that in the numerical modeling of stress wave propagation in two-dimensional solids a cutting trace will be produced violating the results if the dimensional splitting technique is applied. Therefore, a two-dimensional Riemann problem should be considered in the numerical computation. This paper will present a method dealing with a two-dimensional Riemann problem for high velocity impacts of hydro-elastic-plastic materials, which behave under strong stress conditions as compressible fluids in the volume change and obey the von Mises plastic yield law in the shape changes. The numerical scheme for the treatment of stress waves in such materials is divided into two steps including the flux calculation with the proposed two-dimensional Riemann solver and the function updating. Finally, results of the successful application of the scheme to the shear banding problem are presented.

[1] X. Lin and J. Ballmann, "Numerical Method for Elastic Waves in Cracked Solids. Part 1: Anti-Plane Shear Problems," *Arch. Appl. Mech.*, vol 63, pp. 261–282, 1993.

[a] Supported by the Deutsche Forschungsgemeinschaft under Grant No. Ba 661/12-1.

Some Critical Phenomena for Quasilinear Hyperbolic-Parabolic Systems

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We plan to survey recent progress on singular behaviors of nonlinear waves for quasilinear hyperbolic-parabolic systems. These behaviors are due either to partial dissipations, nonstrict hyperbolicity, boundary or other effects. Specific physical examples will be illustrated.

Uniform Third Order Entropy Convergent Scheme for Convex Scalar Conservation Laws

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An entropy condition and a total variation boundedness (TVB) of solutions of convex, scalar conservation laws are enforced by a One-Sided Lipschitz Condition (OSLC), which physical solutions satisfy. The first order Godunov and Lax-Friedrichs schemes, and the second order scheme in [1] are consistent with this OSLC. Hence they are entropy convergent schemes. A uniform 3rd order accurate scheme is introduced here, which is consistent with the OSLC, and hence is an entropy convergent scheme. Numerical experiments on systems of conservation laws were implemented, and excellent results obtained.

[1] Y. Brenier and S. Osher, "The Discrete One-Sided Lipschitz Condition for Convex Scalar Conservation Laws," *SIAM J. Num. Anal.*, vol. 25, pp. 8–23, 1988.

Multidimensional Hyperbolic Systems with Degenerate Characteristic Structure

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We introduce a class of 2×2 quasilinear hyperbolic systems, which we call partially aligned, whose characteristic structure degenerates into a pair of curves through every point. We remark on some of their basic properties and discuss shock formation for a subclass of these systems, both analytically and numerically.

On a System of Nonstrictly Hyperbolic Partial Differential Equations

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Presented by Peizhu Luo

In [1] we studied the Cauchy problem of the system

$$\begin{cases} u_t + (u^2)_x &= 0 \\ v_t + (uv)_x &= 0 \end{cases} \tag{1}$$

with special initial conditions
In the present paper we investigate the general Cauchy problem of the system

$$\begin{cases} u_t + (u^2)_x &= 0 \\ v_t + \lambda(uv)_x &= 0 \end{cases} \qquad \lambda > 0 \tag{2}$$

Instead of considering (2) we consider the nonconservative system

$$\begin{cases} u_t + (u^2)_x &= 0 \\ w_t + \lambda u w_x &= 0 \end{cases} \qquad \lambda > 0 \tag{3}$$

where $w(x,t)$ is the potential of v, i.e.,

$$w(x,t) = \oint_{(0,0)}^{(x,t)} v\,dt - \lambda uv\,dx,$$

$$w_x = v, w_t = -\lambda uv$$

We consider the solution of (3) as the limit of the solutions of the viscosity system

$$\begin{cases} u_t^\epsilon + u^\epsilon u_x^\epsilon &= \epsilon u_{xx}^\epsilon \\ w_t^\epsilon + u^\epsilon w_x^\epsilon &= 0 \end{cases} \tag{4}$$

and satisfying the integral identity

$$\int_0^\infty \int_{-\infty}^{+\infty} (u\varphi_t + \frac{u^2}{2}\varphi_x) dx dt = 0, \tag{5}$$

$$\int_0^\infty \int_{-\infty}^{+\infty} (\psi_t + u\psi_x) dw(x,t) dt = 0, \tag{6}$$

where $\varphi(x,t)$, $\psi(x,t) \in C_0^\infty(R_+^2)$, and the integral (6) is understood in the sense of Lebesgue-Stieltjes integral.
As an application we can easily solve the Riemann problem of the system (2) in explicit form which in general involves δ-functions. This gives supplement to [2], [3].

[1] P. Lou and Z. Ding, "The Initial Value Problem of a System of Conservation Laws," *Acta Math. Scientia*, vol. 13, pp. 111–120, 1993.

[2] A. Forestier and P. Le Floch, "Multivalued Solutions of Some Nonlinear and Nonstrictly Hyperbolic Systems," Internal Report No. 212,, CMAP, Ecole Polytechnique, Palaiseau, France, 1989.

[3] D. Tan, T. Zhang, and Y. Zheng, "Delta-Shock Waves as Limits of Vanishing Viscosity for Hyperbolic Systems of Conservation Laws," to appear.

Spherically Symmetric Motion of Isothermal Gas Surrounding a Solid Ball

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Presented by Tetu Makino

We study the initial boundary value problem

$$\rho_t + u\rho_r + \rho u_r + \frac{N-1}{r}\rho = 0,$$

$$\rho(u_t + uu_r) + (a^2\rho)_r = 0 \qquad (1 \le r),$$

$$u_{|r=1} = 0, \quad \rho_{|t=0} = \rho^0(r), \quad u_{|t=0} = u^0(r).$$

Here $N \ge 1$ is the spatial dimension. By transforming the problem to the Lagrangian coordinates, we apply the method by Glimm and Nashida for $N = 1$. The initial condition are supposed to satisfy

$$\delta_0/(1+x)^{1-\epsilon} \le v_0 \le C_0, \quad |u_0|, \quad T.V.u_0, \quad T.V.v_0 \le C_0,$$

where $0 \le \epsilon \le 1$ and

$$x = \int_1^r \rho r^{N-1} dr \qquad \text{and} \qquad v = 1/\rho r^{N-1}.$$

Since $\inf v_0 = 0$, the C-F-L condition cannot be satisfied for the usual uniform mesh. We present a non-uniform mesh method.

On the Hydrodynamic Model for Semiconductors and Relaxation to the Drift-Diffusion Equation

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Presented by Pierangelo Marcati

We investigate the relaxation problem for the hydrodynamic isentropic Euler–Poisson system, when the momentum relaxation time tends to zero. Very sharp estimates on the solutions, independent of the relaxation time, are obtained to establish the compactness framework.
Namely we consider the 1D system:

$$\begin{aligned} n_t + j_x &= 0 \\ j_t + \left(\frac{j^2}{n} + p(n)\right)_x &= nE - j/\tau \\ E_x &= n - b(x) \end{aligned}$$

for all $(x,t) \in \Sigma = \mathbb{R} \times [0, \infty)$, $\tau > 0$: where n is the electron density, j is the electron current density, E the (negative) electric field and $b \in L^1$ is the density of the fixed (positively charged) back-ground ions. The pressure-density relation is given by $p(n) = \frac{1}{\gamma} n^\gamma$, $1 < \gamma \le 5/3$. All the physical constants are normalized to 1.
We introduce the scaled variables: $\mathcal{N}^\tau(x,s) = n\left(x, \frac{s}{\tau}\right)$, $\mathcal{J}^\tau(x,s) = \frac{1}{\tau} j\left(x, \frac{s}{\tau}\right)$, and $\mathcal{E}^\tau(x,s) = E\left(x, \frac{s}{\tau}\right)$
We prove, as τ tends to zero, $\mathcal{N}^\tau$ converges to a density profile $\mathcal{N}(x,s)$, strongly in L^p_{loc}, $p < +\infty$, $\mathcal{J}^\tau$ converges weakly in L^2_{loc} to a current profile $\mathcal{J}$, and $\mathcal{E}^\tau$ converges uniformly on compact sets to a limit electric field $\mathcal{E}$. Moreover the limit profiles $(\mathcal{N}, \mathcal{E})$ satisfy the classical drift-diffusion equation:

$$\mathcal{N}_S + (\mathcal{N}\mathcal{E} - p(\mathcal{N})_x)_x = \mathcal{E}_x = \mathcal{N} - b(x)$$

and the limit current is recovered from the usual current relation $\mathcal{J} = \mathcal{N}\,\mathcal{E} - p\,(\mathcal{N})_x$.
In order to deal with weak solutions, we perform our limiting process using the methods of compensated compactness following [1] and [2], where a similar analysis was provided for the porous media flow and the nonlinear heat conduction.
The Cauchy problem was investigated by using the classical fractional step Lax–Friedrichs and Godonov schemes in [4]. The "a priori" L^∞ and energy estimates were not uniform with respect to the relaxation time τ. Therefore to overcome such a difficulty we introduced in [5] a new version of the fractional step method which provides the required bounds. Connections with the Von Neumann pseudo-viscosity are also investigated in [3].

[1] P Marcati and A. Milani, "The One-Dimensional Darcy's Law as the Limit of a Compressible Euler Flow," *J Differential Equations*, vol. 84, pp 129–147, 1990

[2] P Marcati, A. J Milani, and P. Secchi, "Singular Convergence of Weak Solutions for a Quasilinear Nonhomogeneous Hyperbolic System," *Manuscripta Math.*, vol 60, pp. 49–69, 1988.

[3] P Marcati and R. Natalini, "Convergence of the Pseudo-Viscosity Approximation for Conservation Laws." (To appear in *J. Nonlinear Analysis TMA*.)

[4] P. Marcati and R. Natalini, "Weak Solution to a Hydrodynamical Model for Semiconductors: the Cauchy Problem." (To appear in *Proc. Royal Soc. Edinburgh*..)

[5] P Marcati and R. Natalini, "Weak Solutions to a Hydrodynamical Model for Semiconductors and Relaxation to the Drift-Diffusion Equation," preprint, 1993. (Submitted.)

A Mechanism for Multiple Asymptotic Solutions for Systems of Conservation Laws

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Presented by Dan Marchesin

We report progress in understanding the dynamics of multiple asymptotic solutions for certain Cauchy problems of parabolic systems associated with pairs of conservation laws in one spatial dimension. The initial data we consider are continuous interpolations between left and right states for which the Riemann problem has multiple solutions; all shock waves in these solutions satisfy the viscous profile admissibility criterion. The presence of non-classical "transitional" shock waves seems to play an important role in the existence of multiple stable Riemann solutions.

[a]Research supported by FINEP.

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Entropy Corrections in Numerical Simulations of an Ideal Gas with the Euler Equations

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In this work some prescriptions are presented and discussed in order to correct entropy in numerical simulations of an ideal gas, modeled with the Euler equations in situations where entropy is violated. In particular, we analyze the classical prescriptions proposed by Colella and Woodward near the corner in the numerical approximation to the classical Mach 3 wind tunnel with a step flow.

[a] This work was made in collaboration with Rosa Donat (University of Valencia).

[b] This work has been supported by spanish DGCYT (grant PS90-0265), in part by a Grant from the I.V.E.I. and computer time supported in part by ARPA URI Grant ONR-N00014-92-J-1890.

The Piecewise Parabolic Method for Relativistic Hydrodynamics

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Presented by José M$^{\underline{a}}$ Martí

Relativistic hydrodynamics plays an important role in different fields of Physics, as Astrophysics, Cosmology or Nuclear Physics. In Astrophysics, the standard model of extragalactic jets assumes an ultrarelativistic bulk velocity of the plasma involved to account for the apparent superluminal motions measured in large-baseline interferometric radio observations of many active galactic nuclei. Analogously, the description of heavy-ion reactions within the hydrodynamic model also requires ultrarelativistic speeds to simulate the high-energy ion beams.

The pioneering ideas of Wilson and coworkers, developed during the 70's, in extending classical artificial-viscosity finite-difference methods to RHD proved to be very successful in the simulation of mildly relativistic flows. However, situations as those mentioned above, are triggering the development of new techniques able to overcome the modelling of ultrarelativistic flows. In this direction, relativistic codes based on Godunov-type methods an approximate relativistic Riemann solvers are getting the most encouraging results.

In this talk, we are going to report on the extension to RHD of the Piecewise Parabolic Method (PPM) of Colella and Woodward, used extensively in classical (Newtonian) simulations. PPM is a well-known high-order extension of Godunov's method. Besides the use of an exact Riemann solver, the key ingredients responsible of the accuracy of PPM are the parabolic interpolation of variables inside numerical cells and the monotonicity constraints and discontinuity detectors that keep discontinuities sharp. Finally, the use of states averaged in the domain of dependence of the interfaces makes PPM second order accurate in time. We are going to present a 1-D relativistic version of PPM in which all the above mentioned ingredients have been properly generalized. In particular we would like to point out the use of an exact relativistic Riemann solver recently developed by the authors.

Detailed results in several tests, including the relativistic version of the interaction of two relativistic blast waves, are shown. Comparison with Godunov's method allows to conclude that the main features of PPM are retained in our relativistic version.

Finally, first results on relativistic jet simulations with a multidimensional relativistic version using PPM reconstruction are presented.

The Effect of a Non-Zero Shock Width on Wave Propagation in Two Dimensions

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Presented by Ralph Menikoff

It is known that a numerical shock width in one dimension can lead to a localized entropy error when waves interact. The error does not dissipate in time and its L^∞ norm does not vanish under mesh refinement. In two dimensions the physical wave width can affect its propagation. An example occurs for detonation waves and is known as the diameter effect. The effect is due to reaction zone dynamics; the competition between a source term for the release of chemical energy and a geometric source term due to front curvature. When the reaction zone is underresolved, the effective reaction zone dynamics leads to mesh dependent numerical results. The mesh dependence is due to an artificial numerical curvature effect. An explanation of this effect is given based on the conservation laws. Because of the non-zero wave width, the standard Hugoniot jump conditions must be modified. Correction terms are proportional to the wave width. It follows when the width is proportional to the cell size, as occurs in shock capturing schemes, that the curvature effect is mesh dependent. A similar modification of the jump conditions can be expected whenever there are multiple length scales determining the dissipation which gives rise to the wave width. As a consequence, the wave curve depends on the local front curvature and the divergence of the velocity field in the tangent plane in addition to the usual state variables. Furthermore, this modification of the wave curve affects two dimensional wave patterns. For example, it is known that the standard Mach wave pattern does not occur for detonation waves. The length scales affecting the wave curve may either be physical or artificial in nature. Artificial length scales lead to numerical errors which may not vanish under mesh refinement.

[a]Supported by U. S. Department of Energy.

Parallel, Mixed Finite Element Approximation for the Porous Media Flow

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An efficient, parallel algorithm for numerical solution of the pressure equation in porous media flow problem is considered. We use lowest order Raviart-Thomas mixed finite elements to obtain high accuracy of the flux approximation, and to ensure mass conservation.
A parallel version of the algorithm was obtained using the non-overlapping domain decomposition method proposed by Glowinski and Wheeler. For this method we propose an quasi-optimal preconditioner, for the subdomain interface problem. Numerical experiments from a parallel implementation on an Intel iPSC/860 hypercube and Paragon computers are presented which show the method to be both efficient and scalable. The numerical examples also confirm the theoretical bounds of the condition number of the interface operator.

Mass Lumping Edge Elements in Three Dimensions

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Presented by Peter Monk

We shall present a dispersion analysis of higher order, mass lumped, edge finite element methods for discretizing Maxwell's equations in three space dimensions. In these methods the electric field is discretized using Nédélec's curl conforming family of edge elements (we shall consider in detail the linear and cubic elements). In order to apply rapid explicit time stepping procedures we use a mass-lumping scheme based on the use of anisotropic quadrature formulae consisting of tensor product Gauss and Gauss-Lobatto rules. Using this technique, it is possible to construct fully discrete, high order in space and time, schemes for approximating the Maxwell system.
Despite the fact that each component of the electric field is approximated in an anisotropic fashion the dispersion relations for the scheme (of any order) can be derived analytically. We shall show that provided an orthogonal grid is used, the dispersion relations for the three dimensional scheme are given by the sum of the dispersion relations for an appropriate mass lumped discretization of the one dimensional wave equation analyzed by Tordjman. Thus the accuracy and stability properties of the scheme for Maxwell's equations can be analyzed easily.
Using simple numerical examples, we shall show that the mass-lumped finite element method offers the possibility of a rapid and phase accurate solution of the Maxwell system.

[a] Research supported in part by AFOSR.

Multidimensional Method of Transport for the Shallow Water Equations

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A truly two-dimensional scheme based on a finite volume discretization on structured meshes will be developed for solving the shallow water equations. The idea of the scheme is borrowed from the method of transport which has been developed by M. Fey to solve the Euler equations for compressible gas [1] In contrast to the Euler equations the flux of the shallow water equation is not homogeneous. We cannot take the eigenvectors of the Jacobi matrix of the flux to compute the partition of the waves C, C^- and U so that we are obliged to modify them.
We had to include source terms and appropriate boundary conditions into the program to give it the ability to simulate river flow or flow in water reservoirs. Some examples will be presented.

[1] M. Fey, "Ein Echt Mehrdimensionales Verfahren zur Lösung der Eulergleichungen," Dissertation, ETH Zürich, 1993.

[2] R. J. LeVeque, *Numerical Methods for Conservation Laws*, Birkäuser, Basel, 1990.

On a Class of Evolution-Galerkin Methods for Low Reduced Frequency Problems

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Presented by K.W. Morton

An important class of hyperbolic problems, of great interest to design engineers, is that of so-called low reduced frequency problems, typified by aircraft flutter, turbomachinery fan/blade interaction, floodwave flows, tidally-driven dispersion, etc.
In this paper we shall first consider the effectiveness of standard unsteady finite difference schemes for these problems. Then we shall consider a formulation of various Evolution-Galerkin methods akin to that used by Godunov; that is, in a finite volume framework it is time integrals of the boundary fluxes, i.e., the inter-cell fluxes, that the evolution operators are used to approximate. The most relevant difference schemes are the box, Crank-Nicolson and Roberts-Weiss angled derivative schemes; and we shall show how useful combinations of these schemes are naturally generated by the new RCFVM (Recovered Characteristic Finite Volume Method) schemes.
A sequence of increasingly demanding model problems will be used to illustrate both analytically and numerically the properties of these finite volume schemes compared with their related finite difference counterparts. The eventual objective of the method development is a scheme for use on unstructured, three-dimensional tetrahedral grids which is a natural development of those presently being used successfully for steady compressible flow problems.

Deformation and Fracture of Solids under Dynamic Loading Conditions

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Continuum mechanics analyses of deformation and fracture in solids subject to rapid loading will be discussed. An overview of the continuum formulation will be given and current areas of research will be noted. The discussion of fracture centers on a framework where the initial-boundary value problem formulation allows for the possibility of a complete loss of stress carrying capacity, with the associated creation of new free surface. No ad hoc failure criterion is employed and fracture arises as a natural outcome of the deformation history. For ductile solids, localization, in the sense of a deformation pattern involving one or more intense deformation bands, can play an important role in the failure process. The discussion will emphasize issues of length scales, size effects and the convergence of numerical solutions.

Aeroelastic Computations of Wings

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Presented by D. Nellessen

The aeroelastic behaviour of a wing is calculated using SOFIA (SOlid-Fluid-InterAction)[1] For the computation of the flowfield the well known code INFLEX[2] is used which solves the hyperbolic Euler equations. The linear elastic structure is assumed to be quasi-one-dimensional. ODISA - a method based on characteristic theory - is used for the calculation of the structural deformations. Both solvers are coupled in the time domain.

INFLEX solves the unfactored, conservative form of the time dependent, implicit Euler equation by a relaxation method. A nonlinear Newton method in connection with a point Gauss-Seidel algorithm is employed. The flux differences are calculated via a characteristic extrapolation scheme based on a Godunov- type averaging procedure.

Due to the theory of Timoshenko and Flügge the effects of shear flexibility and rotatory inertia are included. Each cross-section of the wing has three translational and three rotational degrees of freedom. The bending motions and the torsional motion are coupled statically and dynamically. Therefore the number of independent variables is reduced by the Ritz-Kantorowitsch- Method. The beam is discretized by isoparametric, two noded elements. A reduced integration scheme for the transverse shear contribution avoids shear locking. The resulting set of ordinary differential equations is integrated by Newmark's method.

[1] S. Schlechtriem, D. Nellessen, J. Ballmann, i"Elastic Deformation of Rotor-Blades Due to BVI," in *17th European Rotorcraft Forum*, 1993.

[2] A. Brenneis and A. Eberle, "Evaluation of an Implicit Euler Code Against Two and Three-Dimensional Standard Configurations," in *AGARD CP-507: Transonic Unsteady Aerodynamics and Aeroelasticity*, Paper No. 10, March, 1992.

Numerical Simulation of Shock Waves in Linear-Elastic Plane Plates with Curvilinear Boundaries and Material Interfaces

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Presented by R.J. Niethammer

A numerical scheme of bicharacteristics is employed to compute stress waves in plane, homogeneous, isotropic, linear-elastic plates under in-plane loading with curvilinear boundaries and material interfaces. In order to keep the CFL-number close to 1 and the curved grid coordinates perpendicular to each other to avoid numerical dissipation and dispersion, overlaid grids are used to discretize the computational domain. This approach is validated by comparing the obtained numerical solution with analytical and experimental results for plates with circular or partly circular boundaries and material interfaces.

[a] Supported by the Deutsche Forschungsgemeinschaft under grant no. Ba 661/12-1.

On Conservation Laws with a Complex-Analytic Structure: Special Solutions and Multi-dimensional Upwinding

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Conservation laws with a complex-analytic structure are simple models of hyperbolic systems of two equations in two space variables. As a prototype, they include the Weyl equation of relativistic quantum mechanics and the complex Burgers equation. We present some special solutions, including crossing shocks, a non-local initial-boundary-value problem, radially symmetric solutions and a Roe-type wave model. Using this wave model, we can apply the genuinely multidimensional fluctuation-splitting schemes to conservation laws with a complex-analytic structure. We compare these schemes with standard dimensional splitting schemes and show a situation where the latter ones misrepresent discontinuities traveling obliquely over a cartesian grid.

[a]Supported by Deutsche Forschungsgemeinschaft, SFB 256.

A Numerical Scheme Resolving Shallow-Water Flow Discontinuity with Bathymetry Source Term

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The approximation of the shallow water model with nonconstant grid bottom involves some problems with consistency and even stability when the water depth becomes thin or zero. The more evident example corresponds to the behaviour of a steady fluid, that is with a zero velocity, with a flat fluid surface and a large variation bottom, and for a thin layer of water the processing by using a classical scheme may provide some moving in the fluid and even some instability, for a thin water height, though the fluid should stay at rest. The processing by using split methods cannot also achieve or maintain steady solutions with an acceptable level of accuracy for they do not preserve the balance between source terms and internal forces. We propose here a well balanced scheme which preserves the balance between source terms and internal forces, and which is adapted to any bottom profile. This technique allows the simulation of many phenomena with environmental applications: dam failure with bathymetry and water flooding over a hill.

Energy and Maximum Norm Estimates for Nonlinear Conservation Laws

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Presented by Pelle Olsson

We have devised a technique that makes it possible to obtain energy estimates for initial-boundary value problems for nonlinear conservation laws

$$\begin{aligned} &u_t + f_x = 0, \qquad x \in (0,1) \quad t > 0 \\ &u(x,0) = \varphi(x). \end{aligned} \tag{1}$$

At the boundaries $x = 0, 1$ we prescribe data $\psi(t)$ for the ingoing characteristics, which are determined by the sign of $f'(u(i,t))$, $i = 0, 1$. The two major tools to achieve the energy estimates are a certain splitting of the flux vector derivative $f(u)_x$, and a structural hypothesis, referred to as a cone condition, on the flux vector $f(u)$. The splitting of f_x is defined as

$$f_x = (f - F)_x + F_x,$$

where $F(u)$ satisfies Euler's inhomogeneous differential equation

$$F'u = -F + f \iff F(u) = \int_0^1 f(\theta u)d\theta.$$

Hence, $f_x = (F'u)_x + F'u_x$. Multiplying eq. (1) by u and using this splitting yields

$$\frac{1}{2}(u^2)_t + (uF'u)_x = 0, \quad F'(u) = \int_0^u f'(v)v dv. \tag{2}$$

As a cone condition we take $\text{sgn}(f'(u)) = \text{sgn}(u)$, which ensures that $\text{sgn}(F'(u)) = \text{sgn}(f'(u))$. Integrating eq. (2) in (x,t) space and applying the characteristic boundary conditions of eq. (1) will thus imply an energy estimate.

This splitting technique generalizes to symmetrizable conservation laws in several space dimensions. For many systems that occur in practice, such as the Euler equations of gas dynamics, it is possible to find a suitable cone condition, and thus an energy estimate follows. The results extend to weak solutions that are obtained as pointwise limits of vanishing viscosity solutions. As a by-product we obtain explicit expressions for the entropy function and the entropy flux of symmetrizable systems of conservation laws. Under certain circumstances the proposed technique can be applied repeatedly so as to yield estimates in the maximum norm. Furthermore, it is possible to derive energy estimates for high-order difference approximations of conservation laws if the split version is used.

[a]This work has been sponsored by NASA under Contract No. NAS 2-13721.

The Scope of the Level Set Method for Computing Interface Motion

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In 1987 (with James Sethian) we devised a numerical method using a fixed Eulerian grid, to capture the motion of interfaces in an arbitrary number of dimensions was developed. The "level set" algorithms handle topological merging and breaking easily, (with no user interface) and accurately capture the formation of sharp gradients and cusps in the front.

Recent advances in the method at UCLA enable us to easily compute the motion of multiple junctions, interfaces for multiphase incompressible flow, and Stefan problems. Merging and reconnection are accurately computed.

Very recently, with E. Harabetian, we have proposed and tested a variant which works well for unstable problems, such as motion of vortex sheets.

Theoretical and numerical results will be presented and future applications will be discussed.

Global Weak Solutions for a Model of Two-Phase Flow

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Consider the Cauchy problem for a model of two-phase flow gas/liquid. If the gas and the liquid are of the same velocity u, then we have

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0 \\ \partial_t(\rho c) + \partial_x(\rho u c) = 0 \\ \partial_t(\rho u(1+c)) + \partial_x(\rho u^2(1+c) + p(\rho, c)) = 0 \\ t = 0; (\rho, u, c) = (\rho_0(x), u_0(x), c_0(x)) \end{cases}$$

Here ρ and ρc, with $0 < \rho < 1$, $c > 0$ are the densities of the liquid and the gas. The equation of state is given by $p(\rho, c) = \frac{k^2 \rho c}{1-\rho}$, where k is a constant.

This model is a strictly hyperbolic system with two genuinely nonlinear fields and one linearly degenerate field of Temple's type. We prove the global existence of weak solutions in time for the initial condition (ρ_0, u_0, c_0) with bounded variation. The only hypothesis is that the total variation of the initial concentration is not enough large. More precisely, let $A_0 = k\sqrt{\frac{c_0}{1+c_0}}$. If $TV(A_0) < 2Inf_{x \in \mathbb{R}} A_0(x)$, then there exists a global weak solution defined for all time. The proof uses still the Glimm's scheme. It consists to study carefully the interaction of simple waves and construct the Glimm's functionals to establish the stability estimations.

About Hyperbolic Equations of the Two-Velocity Hydrodynamics with One Pressure

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Presented by Yu. V. Perepechko

The problem of non-hyperbolicity of reversible equations describing two-velocity motion of gas-liquid media is well-known in mechanics of multiphase media. It is caused exclusively by effects of phases transfer with various velocities and results in appearance of unphysical solutions, that is meaning the instability of the media at relative motion of the subsystems. It is accepted to connect non-hyperbolicity of the dynamic equations with absence of the second pressure (which is independent of the first pressure) for one of phases. Therefore it is necessary for hyperbolicity that the number of independent pressures coincide with the number of independent velocities.

We have offered the two-velocity model of gas-liquid media with one pressure which shows that in reality the difficulties are connected with incorrect introduction of reversible forces of interaction between the subsystems, rather than with absence of the second pressure in the system. The correct description of these forces, which is possible in the framework of used approach, based on the fundamental physical principles, results in the model of dynamics of classical two-velocity media with one pressure (like gas- or vapour-liquid media) having hyperbolicity.

Phenomenological theory of classical two-velocity media was created, developing approach, offered Landau L.D. for description of the hydrodynamics of superfluid helium. Demand of the satisfying of general physical principles: conservation laws of the mass, momentum, energy; Galilean invariance of dynamics equations; and fixing of system's kind by choice of the first law of thermodynamics determines the equations of motion of a classical two-velocity continuum uniquely. In advanced theory the energy isn't divided into potential and kinetic ones, and the entropy isn't regarded to be locally additivity function.

The forces of reaction of the subsystems (as forces of apparent masses) are calculated simultaneously with obtaining of the dynamic equations. These forces depend on square of velocities and they have to be taken into consideration when high-velocity flows are investigated. Just their self-agreed definition provides the hyperbolicity of received reversible equations.

Phenomenological equations obtained is possible to be interpreted as the dynamic equations of classical two-velocity continuum, as far as, the entropy is transferred by both components in difference from the Landau's theory.

The set of the dynamic equations is closed by the equation of state of a two-velocity medium, which is obtained in linear approximation without assumption about local additivity of entropy.

Perturbed Elastic Waves: Huygens' Principle and Resonances

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Presented by G. Perla Menzala

We consider smooth solutions of a perturbed system of linear elasticity in 3-D and concentrate our attention on two questions: 1) Is it possible that all such perturbed elastic waves propagate on spherical shells? 2) What can we say about the resonances associated with the perturbed elastic system? Concerning the first question we prove that the answer is negative for a suitable class of perturbations. Our analysis is based on estimates over characteristic cones and a "trace type" theorem suitably adapted for our needs. For question 2) we prove that the resonances form a discrete set in the complex plane and depend continuously on the perturbation.

Theoretical and Numerical Results on Fluctuation-Splitting Schemes

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First, this lecture will be devoted to explain the so-called "fluctuation-splitting schemes" (FSS in short) for solving free advection equations on general triangular grids, these schemes have been introduced recently by DeConninck, Roe. Sildikover Struijs, . . . We will show that they converge strongly in a classical finite volumes sense, although a finite difference version can be derived, with a rate convergence.
In a work with Y. Qiu and B. Stoufflet, we extend these schemes to treat the compressible Euler Equations using the same ideas as in kinetic schemes. Roughly speaking the FSS reduces the problem to a generalized Riemann problem with three states, and the wave decomposition can be naturally performed using the Boltzmann approach. Entropy and stability properties can be rigorously proved for this scheme.
Numerical results show that this approach, although it is only first order accurate, gives comparable or better results than first order finite volumes methods.

The Numerical Solution of a Hyperbolic System with Relaxation Arising in Bioelectromagnetics

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The equations modeling the interaction of pulsed electromagnetic fields with dispersive dielectrics, such as human tissue, consist of a set of hyperbolic partial differential equations (Maxwell's equations) coupled to ordinary differential equations which model the dispersion. The hyperbolic system to be solved is of the form $u_t + Au_x + Bu_y + Cu_z = \frac{1}{\tau_{min}}S(u)$, where $u = (H, E, P)^T$ is the vector of electromagnetic fields, τ_{min} is the smallest relaxation time of the dielectric, and $S(u)$ is the source matrix which represents the dispersion by coupling E and P. Numerical solutions of this coupled system provide input to the development of safety standards for human exposure to pulsed fields. Experimental data for media representative of tissue indicates that the o.d.e. aspect of the problem is very stiff, i.e., $\tau_{min} \ll 1$ while the computed solution evolves on much longer timescales.

I will examine a semi-implicit numerical method for the relevant hyperbolic system with relaxation that is based on staggered-grid finite-differences for the p.d.e. component and on a simple A-stable approach for the o.d.e. component. The severe discretization requirements of this method will be explained with an asymptotic analysis of the coupled p.d.e.-o.d.e. system based on an equation which displays the wave hierarchies in the dielectric: $\partial_t(E_{tt} - c_o^2 E_{xx}) + \frac{a}{\tau_{min}}(E_{tt} - c_1^2 E_{xx}) = 0$, where $\frac{a}{\tau_{min}} \gg 1$. An alternative, more "physical," finite-difference scheme will be presented along with arguments for its superiority that are partly based on the analysis.

[a]This work was supported by contract F41624-92-D-4001 with the USAF Armstrong Laboratory.

Visco-Plastic Relaxation: Convergence and Localization [a]

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A visco-plastic shear-strain softening constitutive model is introduced as a regularization of an unstable elasto-plastic softening model. Mathematical analysis demonstrates that, in spite of softening, the solution of the visco-plastic model has bounded energy; furthermore, the solution (in particular the stress) converges in an appropriate sense, to the solution of the elasto-plastic model. A computational scheme based on a high-order Godunov method is then used to compute numerical solutions to the visco-plastic problem.

[a]Research supported by the National Science Foundation under Grant DMS-9201062, which includes additional funds from the Air Force Office of Scientific Research

Conservation Principles for Elasto-Plastic Materials

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In this talk we shall describe a new formulation of the continuum equations modeling elasto-plastic materials. The equations form a system of conservation laws [2], which naturally permits discontinuous solutions. This offers advantages for both computation and analysis.

The decisive advantage of conservative numerical schemes is well established. We have developed a high-resolution, conservative Eulerian numerical scheme that incorporates material interface tracking and have applied it to metal plate impact problems [4]. One outcome of this work [3] is to correct the value of a fundamental material parameter that had been overestimated, by a factor of three, using standard codes.

We have also analyzed the structure of a shear band in a metal, one of the principle wave modes in elasto-plastic flow. Asymptotic analysis [1] shows that, in a fully developed shear band, the stress/strain-rate relation is controlled by the balance between heat conduction and the heat generated by plastic work. This picture leads to a tracking algorithm for the effective computation of shear bands.

[1] J. Glimm, B. Plohr, and D. Sharp, "A Conservative Formulation for Large-Deformation Plasticity," *Appl. Mech. Rev.*, vol. 46, pp. 519–526, 1993.

[2] B. Plohr and D. Sharp, "A Conservative Formulation for Plasticity," *Adv. Appl. Math.*, vol. 13, pp. 462–493, 1992.

[3] F. Wang and J. Glimm, "Models for Rate-Dependent Plasticity," Report No. SUNYSB-AMS-94-06, June, 1994.

[4] F. Wang, J. Glimm, J. Grove, B. Plohr, and D. Sharp, "A Conservative Eulerian Numerical Scheme for Elasto-Plasticity and Application to Plate Impact Problems," *Impact Comput. Sci. Engrg.*, vol. 5, pp. 285–308, 1993.

Fluid Flow Modeling in Manufacturing Processes

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Transport phenomena involving fluid flow, heat transfer and mass transport play an important role in many materials and manufacturing processes. In many applications, the process and product quality can be controlled and improved by developing a better understanding of the transport processes through fluid flow modeling. CFD based computer models are becoming very popular for process simulation. State-of-the-art of modeling and simulation in the areas of low and high pressure crystal growth, and melting/solidification will be presented.

Reservoir Simulation by Front Tracking

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Presented by N. H. Risebro

We will present the "Oslo approach" to front tracking, applied to reservoir simulation. The ideas behind the front tracking will be discussed, and then we will show how front tracking methods can be used as numerical methods for some models of flow in porous media, see [1].

The front tracking method has been used as a basis for a commercial reservoir simulator, and we will give several examples taken from "real" reservoir studies, where front tracking has been used to model the flow of water and hydrocarbons. In many cases, particularly in two dimensional models of two phase flow, experience has shown front tracking to be superior to methods based on finite differences with regards to accuracy vs. CPU-time.

[1] F. Bratvedt, K. Bratvedt, C. Buchholz, H. Holden, L. Holden, and N.H. Risebro, "A New Front Tracking Method for Reservoir Simulation," *SPE Reservoir Engineering*, pp. 107-116, 1992.

Inverse Scattering for an Integro-PDE Associated with Electromagnetic Dispersion

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This talk is about time-domain inverse scattering for electromagnetically dispersive media that are flat, and whose properties vary only with depth. Examples include soils and tissues These media are represented by an integral kernel $g(x,t)$:

$$E_{xx} - E_{tt} + \int_0^t g(x, t-s)E(x,s)ds = 0$$

The kernel $g(x,t)$ is to be determined from data concerning the electric field $E(x,t)$ at a boundary $x = 0$. In particular, the goal is to use time-domain reflection data $R^\theta(t)$, as a function of the angle of incidence θ, to determine the two-variable function $g(x,t)$ that characterizes the medium. The talk will review progress in this work.

[a] This work was funded under USAF contract F41624-92-D-4001 and was sponsored by the HQ Human Systems Division at Brooks Air Force Base. The work was done at Brooks AFB under a program administered by Operational Technologies Corporation.

An Approximate Riemann Solver for Magnetohydrodynamics (That Works in More than One Dimension)

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Presented by P. L. Roe

An approximate Riemann solver has been developed for the governing equations of ideal magnetohydrodynamics (MHD). The Riemann solver is a Roe-type solver, with an eight-wave structure, where seven of the waves are those used in previous work on upwind schemes for MHD, and the eighth wave is related to the divergence of the magnetic field. The structure of the eighth wave is not immediately obvious from the governing equations as they are usually written, but arises from a modification of the equations that is presented in the talk. The addition of the eighth wave allows multi-dimensional MHD problems to be solved without the use of staggered grids or a projection scheme, one or the other of which was necessary in previous work on upwind schemes for MHD. Topics covered in the talk include:

- The structure of the eight-wave Riemann problem;
- The derivation of the Roe-average state for ideal MHD;
- Normalization of the eigenvectors to avoid singularities.

A test problem made up of a shock tube with rotated initial conditions is solved to show that the two-dimensional code yields answers consistent with the one-dimensional methods developed previously. In addition, results of calculations of the interaction of the solar wind with a comet on an adaptively refined mesh will be shown.

Multidimensional Upwinding—the Wave of the Future?

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Upwind differencing, in one version or another, is irresistably natural in one space dimension, but loses some of its compulsion in higher dimensions. There are many possible ways to attempt the generalization, but almost all of those that have been successfully implemented follow a dimension-by-dimension approach that gives unsatisfactory results in separated shear layers and at stagnation points. After a critique of this method, we present new decompositions based on the following elements.

1. A computational method, derived from desirable properties of the modified equation, which comes in hyperbolic and elliptic flavors.
2. Decomposition of the steady state operator into hyperbolic (scalar) and elliptic (2x2) subsystems.
3. *Preconditioning of the time-marching* to decouple the solution strategies for each subsystem.
4. A parallel computing strategy that minimizes indirect addressing and message-passing.

The Three-Dimensional Riemann Problem for the Linearized Gas Dynamic Equations

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Presented by Carole Rosier

We explicitly solve the three-dimensional Riemann problem for the linearized gas dynamic equations on a structured mesh. We use the Fourier transform to derive formulae in the Fourier space. Then the inverse Fourier transform gives the pressure P and the three components of the velocity.

Some Hyperbolic Problems in Industrial Applications

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In this talk, we shall discuss some novel hyperbolic equations that have arisen in industrial problems. We shall explain the physical basis for the models that are expressed by these equations, we shall consider some mathematical oddities that complicate the formulation of well-posed problems, and we shall discuss the solution and applications of the equations.

The first class of equations we shall discuss is arise from a simple model of plasma etching, a method used for boring contact holes as part of the manufacturing of microelectronic devices. The equations are scalar conservation laws with non-convex flux functions. The problems are moving boundary problems, and the non-convexity of the flux functions provides some complications in establishing proper moving boundary conditions. We shall discuss these issues, as well as current applications and research on numerical methods for these equations.

The second class of equations we shall discuss arise in the modeling of batch crystallization. These are evolution equations with a generally hyperbolic character, but the presence of non-local terms - a delay term or an integral expression - make them non-standard. They are equations for the evolution of a population density function, and they are non-linear in an unusual way; the coefficients in apparently linear equations are nonlinear functions of the moments of the population density. We shall discuss the qualitative behavior of such equations, including some asymptotic results, and we shall discuss their use in the modeling of the growth of silver halide crystals for use in photographic film.

Small Correction to the Theory of Supersonic Inviscid Gas Flow About a Wedge

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Problems of flow about a wedge or a cone have been under theoretical and experimental study for more than sixty years. However, the modern literature on gas dynamics provides no clear explanation of the role of the so called " strong shock" in the theory of such a flow.

Even the best monographs, text-books and research papers contain erroneous assumption that both known self-similar flows, i.e., that with a weak shock and that with a strong attached to wedge, are, at least formally, solutions to the same physical and mathematical problem of steady free flow of an inviscid non-heat-conductive gas about an infinite wedge (or cone).

However, only the self-similar flow with a week shock is indeed a proper free flow about an infinite wedge (or cone), whereas the flow with a strong shock is related to the different and more complex physical problem of gas flow about the wedge. This circumstances allow us to correct the conventional theory of the flow about a wedge (or cone).

Porous Media Flow as the Limit of a Nonstrictly Hyperbolic System of Conservation Laws

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We show that the weak solutions of the quasilinear hyperbolic system

$$\begin{cases} \epsilon u_t^\epsilon + \left(\epsilon\left(n+\frac{1}{2}\right)(u^\epsilon)^2 + f(v^\epsilon)\right)_x = -u^\epsilon \\ v_t^\epsilon + (u^\epsilon v^\epsilon)_x = 0 \end{cases} \tag{1}$$

converge, as ϵ tends to zero, to the solutions of the reduced problem

$$\begin{cases} u + f(v)_x = 0 \\ v_t + (uv)_x = 0 \end{cases}$$

so that v satisfies the nonlinear parabolic equation

$$v_t - \left(f(v)_x v\right)_x = 0.$$

The basis for studying the system of the problem (1) depends on the following fact: our solutions are asymptotic profiles of the solutions of the system

$$\begin{cases} U_s + \left(\left(n+\frac{1}{2}\right)U^2 + f(V)\right)_y = -U \\ V_s + (UV)_y = 0. \end{cases} \tag{2}$$

In fact, the rescaling

$$\begin{cases} u(y,s) = \sqrt{\epsilon}U\left(x/\sqrt{\epsilon}, s/\epsilon\right) \\ v(y,s) = V\left(x/\sqrt{\epsilon}, s/\epsilon\right) \end{cases} \tag{3}$$

for $\epsilon > 0$ transforms the problem (2) into the problem (1).

We assume that $\epsilon > 0$, $(x,t) \in \mathbf{R}_x \times \mathbf{R}_t^+$, $f \in C^2(\mathbf{R})$ such that

$$f(v) = kv^{2n} + o\left(|v|^{2n}\right)$$

and

$$\begin{cases} \xi f'(\xi) > 0 & \text{for } \xi \neq 0 \\ f''(\xi) > 0 & \text{for } \xi \neq 0. \end{cases}$$

A study of this kind is useful to understand relations between the theory of quasilinear hyperbolic equations and degenerate diffusive phenomena.

We extend the existence theory for nonstrictly hyperbolic 2×2 system investigated in some previous papers. The limiting procedure is then carried out by using the theory of compensated compactness. Finally we obtain the existence of Lyapounov functionals for the limit parabolic equation as weak limit of the convex entropies as ϵ tends to zero for the corresponding hyperbolic system.

PDE's, Crime and Video Tapes (Applications of Nonlinear Numerical Analysis to Investigative and Trial Image/Video Processing)

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Rapid proliferation of affordable video technology (recording, storage and transmission)redefined industrial and public security systems. It furnished law enforcement, judicial and legal community with a powerful new tool. In practice, to make use of collected and stored video information, images are being processed by computers. Computational image processing (CIP)is a new branch of applied mathematics that finds rigorous mathematical solutions to problems associated with restoration and analysis of decayed (e.g. cluttered and blurry) images and movies. Recent advances in fast numerical algorithms for nonlinear problems have paved a new approach to image processing. The Total Variation (TV) based image restoration technique was developed by researchers at Cognitech, Inc. to enable accurate numerical reconstruction and analysis of information containing image features. While traditional image processing algorithms introduce blurring and ringing (oscillatory) artifacts, the TV-based techniques are sharp and non-oscillatory. The TV based image restoration technique geometrically amounts to minimizing the length of the level sets of a reconstructed image, subject to image degradation constraints. In practical applications, one assumes a space-varying blurring kernel and multiplicative noise. In TV image restoration, the solution is obtained by solving a time-dependent, nonlinear partial differential equation (PDE) on a manifold that satisfies the degradation constraints.
We will demonstrate applications of the TV based and other modern CIP techniques to video evidence from actual criminal and civil investigations (e.g., investigation of homicide, assault during LA riot, insurance fraud).

On the Correctness of Cauchy Problem to One-Dimensional Models of Isentropic Gas Dynamics

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Let us consider the following system of two conservation laws which represent the equations of isentropic gas dynamics
The first one is in *Eulerian* coordinates

$$\begin{array}{rcll} \rho_t & + & m_x & = 0 \\ m_t & + & (m^2/\rho + P(\rho))_x & = 0, \quad (t,x) \in R_+ \times R \end{array} \tag{1}$$

with initial data

$$\begin{array}{rcl} \rho(0,x) & = & \rho_0(x) \in BV(R) \cap L^\infty(R), \quad \rho_0 > 0, \\ m(0,x) & = & m_0(x) \in BV(R) \cap L^\infty(R), \end{array} \tag{2}$$

where $\rho > 0$ is density ; $m \equiv \rho u$, u is velocity ; $P(\rho)$ is pressure, $P' > 0$, $P'' > 0$ as $\rho > 0$.
The second one is in *Lagrangian* coordinates

$$\begin{array}{rcll} v_t & - & u_h & = 0 \\ u_t & + & A(v^{-\gamma})_h & = 0, \end{array} \tag{3}$$

where $v = 1/\rho > 0$, h is the Lagrangian coordinate, $A = const > 0$, and $\gamma > 1$.
Concerning the problems (1) and (2) by means of Glimm's scheme (but not providing small variations and maximum moduli of unknowns), one obtains the following theorem.
THEOREM 1. Suppose $\rho \geq const > 0$ for any $x \in R$, $t > 0$. Then there exists the generalized solution to the problem (1), (2) in the sense of distributions for any $t > 0$.
The proof is based on the maximum principle and on smoothness of the curves in the phase plane representing solutions to the Riemann problem. So, in general, it can be transferred to systems of $n > 2$ conservation laws.
THEOREM 2. The Cauchy problem for system (3) in the case $\gamma = 3$ is ill-posed in the sense of distributions. Namely, there exists an L_1-converging sequence of initial data such that the corresponding sequence of smooth solutions to (3) is nonconvergent in the sense of distributions.
This fact is in connection with the presence of a vacuum state in (3).

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The Optimal Convergence Rate for Monotone Finite Difference Schemes [a]

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In the case of a single scalar hyperbolic conservation law

$$\partial_t u + div_x f(u) = 0 \quad u(\cdot, 0) = u_0,$$

it is known that monotone conservative and consistent finite difference schemes on uniform grids converge to the entropy solution at a rate of $O(\sqrt{\Delta x})$, where Δx is the meshsize. For a linear equation, i.e. when the flux f is linear, it was proved that this convergence rate is optimal. For nonlinear equations, there are examples (a single shock wave or a single rarefaction wave) for which the convergence rates are higher ($O(\Delta x)$ for the shock and $O(\Delta x\,|ln\,\Delta x\,|)$ for the rarefaction). This leads to the question of whether convergence rates higher than $O(\sqrt{\Delta x})$ are possible for monotone schemes applied to nonlinear problems, with general initial data in $L^1 \cap L^\infty \cap BV$. We prove that this is not possible by constructing examples that show the optimality of the $O(\sqrt{\Delta x})$ rate for general *nonlinear* fluxes and initial data in $L^1 \cap L^\infty \cap BV$ or in $L^\infty \cap BV$.

[a]This research was supported by the Office of Naval Research Contract N0014-91-J1343.

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Approximate Jump Conditions for Hyperbolic Systems

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Hyperbolic systems in nonconservation form are now widely used in the mathematical modeling of two-phase fluid flows. In the same manner as for conservative hyperbolic systems, we expect the formation of shocks in solutions after a finite time, even for smooth initial data. However, unlike what happens for conservative systems, one does not dispose of any explicit jump relations to characterize shock waves solutions of a system in nonconservation form and even the definition of shock waves needs to be made more precise. Recalling that the physical system modeling the two-phase flow is a convection-diffusion system, we define the shock wave solution of the first order system extracted from the second order system as the limit, when the diffusion is neglected, of traveling waves solution of the full system. Next, it turns out that the coefficients in front of the terms in nonconservation form are small and we take advantage of this to write explicit approximate jump conditions for the shock waves solutions defined above; our approximate conditions are obtained as an expansion of exact but nonexplicit jump conditions. Furthermore the approximate conditions can be written as a perturbation in nonconservation form of some explicit jump conditions in conservation form. We successfully use the approximate jump conditions to write a Roe-type numerical scheme for the solution of a hyperbolic system in nonconservation form modeling two-phase fluid flows.

Riemann Problem Solutions of Codimensions 0 and 1

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Presented by Stephen Schecter

For a system of two conservation laws in one space dimension, we consider Riemann problem solutions that are stable to perturbation of the Riemann data. In other words, if the left state, right state, and flux functions are perturbed, there is a new Riemann problem solution that contains the same sequence of wave types as the old. Shocks are deemed admissible if the viscous profile criterion is satisfied.

A large (and probably complete) class of Riemann problem solutions that are stable in this sense is identified, some of which contain types of shock waves that have not previously appeared in the literature. For a Riemann problem solution to be in this class, certain restrictions on the order of the wave types must be satisfied, as must an explicit set of nondegeneracy conditions. If one nondegeneracy condition is violated, we obtain a codimension one Riemann problem solution, such as those that occur on U_R-boundaries in Riemann problem solution diagrams in the literature. *There is thus the possibility of a* systematic program to understand codimension one Riemann problem solutions.

I shall report on progress to date on this program. One interesting observation is that the data for a codimension one Riemann problem solution can belong to a boundary between two half-neighborhoods, one in which the data give rise to two nearby Riemann solutions, and the other in which there are none.

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[b]Research supported by NSF, CNPq, FINEP and the U.S. Department of Energy.

[c]Research supported by NSF, ARO and the U.S. Department of Energy.

BV Bounds for a Harten-Lax-Type Approximation

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Presented by Steve Schochet

The existence theory for systems of conservation laws is based on Glimm's random-choice method. Convergence of Godunov's method, in which the random choice is replaced by averaging, remains an open question. Since averaging methods use less information about solutions of Riemann problems, they are simpler to implement numerically. Harten and Lax have proposed a family of intermediate methods that involve averaging over parts of the solution to the Riemann problem, and retain the use of a random choice among these partial averages. They also showed that these methods converge provided that the approximants have uniformly bounded total variation; such a bound was proven by Glimm for the random-choice method. We prove a BV bound for an approximation of Harten-Lax type for the case of pairs of conservation laws. The approximation does not average isolated shock waves, so such a wave propagates undistorted, as in the random-choice scheme. The BV bound is obtained by introducing an additional term into Glimm's interaction potential to account for the effect of partial averaging. A further novelty is that the decrease over time of the potential occurs only for most random choices, unlike the case of Glimm's scheme in which the random choice is needed only to obtain consistency.

Particle Code for Solving the Euler Equations in 2D Based Upon a Kinetic Scheme

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Presented by W. Schreiner

For years particle codes for solving the Boltzmann Equation have been heavily used by our group in Kaiserslautern. For an easy coupling of this code to an Euler solver, we developed a particle code for solving the full system of Euler equations based upon a kinetic scheme initially developed by S. Kaniel, S.M. Deshpande and B. Perthame. Our scheme is consistent with the *anisentropic* system of the Euler Equations for a general equation of state and polyatomic gas (i.e. air). The code works on a smoothed Voronoi-Triangulation and is of second order in space. To ensure this we are going to transform *low discrepancy sequences* in four dimensions — generated by the very fast *Kakutani-Transformation* — to a linear approximation of the distribution function by a new transformation.

[a] Research of the third author supported by the Deutsche Forschungsgemeinschaft throughout the Graduiertenkolleg Technomathematik in Kaiserslautern.

Error Estimate for Godunov's Method Applied to Conservation Laws Including Stiff Source Terms [a]

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Presented by H. Joachim Schroll

Hyperbolic systems with source terms arise in many important applications. In this talk we estimate the error of Godunov's method applied to a scalar equation with a general non-stiff source term

$$u_t + f(u)_x = g(u).$$

We use arguments due to Lucier [1] and extend his results. Furthermore, error bounds for straightforward finite difference schemes including operator splitting methods are obtained by comparing to Godunov's method.

The following system with a stiff source term arises in chromatography [3]:

$$\begin{aligned} u_t + f(u)_x &= \frac{1}{\delta}(v - A(u)) \\ v_t &= \frac{1}{\delta}(A(u) - v). \end{aligned} \tag{1}$$

Here f and A are given, increasing functions, and $\delta > 0$ is the relaxation time.

Based on δ-independent estimates of the entropy solution of (1), which are established in [2], the error of Godunov's method applied to (1) can be estimated. We derive an L^1-error bound of order $\Delta x^{1/2}$, independent of the relaxation time. Here Δx is the mesh size.

The estimates of order $\Delta x^{1/2}$ are optimal, since the corresponding estimates for equations without a source term are known to be optimal.

[1] B. J. Lucier, "Error Bounds for the Methods of Glimm, Godunov and LeVeque," *SIAM J. Numer. Anal.*, vol. 22, pp. 1074–1081, 1985.

[2] A. Tveito and R. Winther, "On the Rate of Convergence to Equilibrium for a System of Conservation Laws Including a Relaxation Term," preprint, 1994.

[3] G. B. Whitham, *Linear and nonlinear waves*, John Wiley & Sons, 1973.

[a] This research has been supported by the Norwegian Research Council (NRF).

Some Applications of Wave Hyperbolic Modeling and Wave Decomposition

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The hyperbolic initial boundary value problem in E_n is considered for a layer between two hypersurfaces with one coordinate hyperline orthogonal to these hypersurfaces. The thickness of this layer is assumed to be much less than a characteristic in-hypersurface length. Expanding field functions in power series with respect to a middle hypersurface yields a degenerate infinite system for functions depending now on $n-1$ coordinates. Reducing this system allows us to obtain many different approximations, i.e. simplified models. The aim is to derive hyperbolic approximations degenerate with respect to one coordinate, i.e. to construct a mapping E_n to E_{n-1} satisfying the condition of limiting correctness, that is, the condition of finite velocity of propagation of disturbances (Selezov, 1969, 1989). A sufficient condition is announced for obtaining hyperbolic approximations: to keep in infinite systems all space-time differential operators up to a given order. This is proved in the case of E_3 considering the elastodynamic problem for the rectilinear layer (Cauchy, 1831; Poisson, 1832). The cumbersome infinite system is split into two independent systems, corresponding to symmetric and asymmetric fields, and two hyperbolic approximations are obtained in both cases including known and new models.

The initial-boundary value problem for inhomogeneous quasi-linear systems of n^{th} order PDE's with unknown variables of spatial coordinate s and time t is considered. After transition to a discrete finite difference model, the matrix operator of convective terms is decomposed by wave modes or wave types. The wave decomposition is based on the evidence that in real structures the wave energy is transported by a discrete set of wave modes in wave guides and/or the certain types of wave motions. In the case when a coupling between the wave modes is not strong, this gives the basis to split the operator and then provide its approximate factorization. The efficiency of the proposed approach is demonstrated by solving the problem of transient motion of an elastic cable system in flow.

Some hyperbolic models obtained as extensions of preliminary known parabolic models are presented and briefly characterized. Examples include the model of kinetic theory (Maxwell, 1867), the model of diffusion (Davydov, 1935), the generalized Smolukhovsky equation (Devis, 1954), the model of a relativistic elastic solid (Bento, 1985), the model of sediment evolution (Selezov, 1982).

[a]This work supported by the American Physical Society and by the Fund for Fundamental Research of the Ukrainian State Committee on Science and Technology.

Liquid-Vapor Flows for Compressible Flows: A Variational Approach

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One considers viscous compressible flows for two-phase fluids. The phases are the liquid and the vapor of the same component. The extensive study about the admissibility of phase boundaries (Slemrod et al.) has shown the unavoidable role of the capillarity. According to Korteweg's theory, capillarity is a contribution of the density gradient to the internal energy $E = e(\rho, \epsilon\nabla_x\rho)$. The two phases correspond to domains where the equilibrium energy $e_0 = e(\cdot, 0)$ coincides with its lower convex envelope. The positive parameter ϵ describes the smallness of capillar effects as well as viscous effects. One studies the asymptotic behaviour of solutions $(\rho^\epsilon, u^\epsilon)$ of the generalized Navier-Stokes equations as ϵ goes to zero, by means of an asymptotic development. Amplitudes are $O(1)$ for ρ and $O(\epsilon)$ for u whereas, the local wavelength (let's speak about the 1-d case for simplicity) is $Y(t,x)\epsilon$. It turns out that Y is a new thermodynamic variable of the flow (and the only new one), which plays the role of an entropy. One gives a variational formulation of Y in terms of the averaged density and internal energy (and usual entropy if relevant). One shows that the macroscopic flow does not depend on the particular viscous law that we may choose. However, studying the stability of the asymptotic development, one shows that the absence of viscosity makes it unstable. Then the behaviour as ϵ goes to zero obeys a different description, with two new thermodynamic variables instead of one (this last part in collaboration with S. Gavrilyuk, Novossibirsk).

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Systems of Fully Nonlinear Partial Differential Equations Derived from Hypoplasticity

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Various properties of systems of fully nonlinear partial differential equations representing simplified hypoplasticity models are discussed. The main simplification is to assume the relation between stress rate and strain rate is independent of stress. Because the equations are homogeneous of degree one, it is straightforward to define shock wave solutions. Admissibility conditions for solutions of the Riemann problem are formulated in terms of a scale invariant system of equations that includes viscosity terms. Similarity solutions of the viscous system are studied using asymptotic analysis as the viscosity approaches zero. For a limited class of equations, it is shown that linearized hyperbolicity of the hypoplasticity equations is necessary and sufficient for the well-posedness of Riemann problem. Examples demonstrate that linearized hyperbolicity is in general not sufficient for the existence and uniqueness of solutions. This issue raises interesting open questions regarding the hypoplasticity equations.

A New Genuinely Two-Dimensional Scheme for the Compressible Euler Equations [a]

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A common approach towards the construction of multidimensional numerical schemes for gas dynamics is dimensional splitting, i.e. extending a one-dimensional scheme to multidimensions on a dimension-by-dimension basis. Schemes created in this way are widely used in practice, although there is concern that they may produce numerical artifacts. Also, the steady-state solvers based on such schemes suffer from poor computational efficiency. These difficulties motivated the search for genuinely multidimensional schemes. Some genuinely two-dimensional advection (scalar) schemes have been available already for several years, however, the extension of these ideas to systems of equations appeared to be a difficult problem.
In this talk, a brief review of some genuinely two-dimensional high-resolution advection schemes will be given. Then a robust genuinely two-dimensional scheme for the compressible Euler equations will be presented. The construction of this scheme is based on the

- observation that the artificial viscosity of some commonly used numerical schemes for the Euler equations in one dimension has a certain very simple structure;
- generalization of the strategy used to construct the scalar advection schemes.

The resulting high-resolution scheme is formulated on triangular meshes and its basic version can be interpreted as a truly two-dimensional extension of the Roe scheme.
One of the important advantages of this method is that Gauss-Seidel relaxation is stable when applied directly to the high-resolution scheme. To our knowledge there is no other high-resolution discretization scheme for the Euler equations in two dimensions that achieves this. The efficiency of the resulting multigrid solver is therefore comparable to what can be achieved in the scalar case.
Some possible extensions of the truly multidimensional approach will be discussed. Numerical experiments will be presented.

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Blow–Up of Solutions of Balance Laws

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Presented by C. Sinestrari

We consider the Cauchy problem for the scalar balance law

$$\partial_t u(x,t) + \sum_{i=1}^{n} \partial_{x_i} f_i(u(x,t)) = g(u(x,t)) \qquad x \in \mathbb{R}^n, t \geq 0. \tag{1}$$

We assume that the initial value is non negative and that $g(0) = 0$. Due to the presence of the source term g, the solution may become unbounded at some critical time $T^* < +\infty$; if this happens, we look for continuations after the blow–up time, which are locally bounded solutions of (1), defined in some open connected set $\mathcal{A}$, with $\mathbb{R}^n \times [0, T^*] \subset \mathcal{A} \subset \mathbb{R}^n \times \mathbb{R}_+$, coinciding with u in $\mathbb{R}^n \times [0, T^*]$.

A way to construct continuations can be described as follows. We choose a sequence $\{g_k\}$ of functions, growing no faster than linearly, converging to g from below, and call u_k the corresponding solution of (1), which is globally defined. It turns out that the sequence $\{u_k\}$ is nondecreasing, and that the limit $U := \sup u_k$ is a continuation in the maximal open connected set $\mathcal{A}$ where it is locally bounded. We call U a *monotone continuation* of the solution u of (1).

In the one–dimensional case we can describe in detail the properties of monotone continuations.

Theorem. *Consider equation (1) with $n = 1$. Suppose that f is strictly convex, that $f(u) \to \infty$ as $u \to \infty$ and that the initial value has compact support. Let u be a solution which becomes unbounded at time T^*, and let U be a monotone continuation of u, defined in the open set $\mathcal{A}$. Then either $\mathcal{A} = \mathbb{R} \times \mathbb{R}_+$, or it has the form*

$$\mathcal{A} = \{(x,t) : t \geq 0, x < \chi($$

where $\chi : [0, \infty[\to \overline{\mathbb{R}}$ satisfies
(i) $\chi(t) \equiv \infty$ for $t < T^*$, $-\infty < \chi(t) < \infty$ for $t \geq T^*$;
(ii) for any $t \geq T^*$, $U(x,t) \to \infty$ as $x \to \chi(t)$ from the left;
(iii) there exists a constant C such that

$$\chi(t) - \chi(s) \leq C(t-s), \ \forall t > s \geq T^*.$$

In particular, $\chi \in BV_{loc}([T^*, \infty[)$.

Using this result, we can give an axiomatic characterization of monotone continuations, which yields an uniqueness result, and allows us to compute explicitly the continuation for some classes of initial data.

Interrelationships Between Viscous and Relaxation Limits for Hyperbolic Systems of Conservations Arising in Gas Dynamics

M. Slemrod

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The speaker will survey investigations into the role of viscosity and relaxation for the hyperbolic systems describing gas dynamics. Both kinetic and continuum theories are discussed. Numerical experiments will also be provided.

Shock Waves and General Relativity

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A general form of obtaining shock-wave solutions to the Einstein equations is outlined. This involves matching two metrics, Lipshitz continuously across a surface of discontinuity of the fluid variables, and involves some geometric conditions. Possible applications to both cosmology, and stellar dynamics will be discussed.

Flame Tracking Via Wave Propagation Methods

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Presented by Vančo Smiljanovski

Associated with the simulation of low-speed flame combustion are two particular difficulties that we address in this work.
The first difficulty arises from the wide separation of scales between the chemically induced length and time scales and the flow scales. In this present work this problem is addressed by considering the flame as a discontinuity. The approach to shock tracking based on high resolution wave propagation methods proposed by R.J. LeVeque has been extended in order to account for the tracking of flames. In one space dimension an underlying uniform grid, on which high resolution shock-capturing methods are applied, is used with additional grid interfaces introduced at appropriate points for tracked flames. Conservative high resolution methods based on the large time step wave propagation approach are used on the resulting nonuniform grid.
The second difficulty is to account for the large heat release and the associated changes of density and flow velocity across the flame surface. To this end we incorporate the exact Riemann-solver that includes a flame discontinuity in the wave propagation algorithm. For the unique solution of the Riemann-problem we prescribe a state dependent laminar burning velocity s_L which also accounts for nonlinear flame acceleration by preheating through compression waves. Furthermore, a turbulent flame surface area increase A, yielding a turbulent burning velocity $s_T = A \cdot s_L$, is introduced in order to simulate turbulence-induced flame acceleration.
As an example we consider a model for the deflagration to detonation transition (DDT). Combustion affects this flow in two ways; complete burning in a turbulent flame with a prescribed time dependent flame surface area increase $A(t)$ interacts with an Arrhenius-type autoignition chemistry in the unburnt gases, which we account for by a standard operator splitting technique. The computation exhibits the generation of acoustic waves and the final transition to detonation through a SWACER-like mechanism, triggered by acoustic waves.

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On Discrete Travelling Shocks of Conservation Laws

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We study the behaviour of finite difference schemes approximating solutions with shocks of conservation laws.
The sharps shocks of the exact solutions of scalar conservation laws are hardly reproduced when those are approximated by finite difference schemes. When a finite difference scheme introduces artificial numerical diffusion, for example the Lax-Friedrichs scheme, we experience smearing of the shocks, whereas when a scheme introduces numerical dispersion, for example the Lax-Wendroff scheme, we experience oscillations on both sides of the shock.
Gray Jennings studied approximation by monotone schemes. These contain artificial viscosity and they are first order accurate ; they are known to be contractive in the sense of any l^p norm. Jennings showed existence and l^1 stability of travelling discrete smeared shocks for such schemes.
Smyrlis, in his dissertation studied similar questions for the Lax-Wendroff scheme without artificial viscosity ; this is a non-monotone, second order accurate scheme. He proved existence, parametrization and stability of stationary profiles for the Lax-Wendroff scheme.
In the present work we study questions of existence and stability of travelling profiles for the the same scheme. Stability is defined in the sense of a suitably weighted l^2 norm. The work relies on examination of the linearized Lax-Wendroff scheme around the travelling shock.

Unsteady Transparent Boundary Conditions for Transonic Flow Problem in Windtunnel

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A flow problem in infinitely long windtunnel with subsonic inlet is considered, flow unsteadiness and local supersonic zones induced by a streamlined body are admitted. In order to bound the computational domain the nonlocal artificial boundary conditions on some front and back crossections are suggested. The conditions are obtained by using unsteady Euler equations linearized around the free-stream uniform flow. To find the analytical expressions for the conditions the auxiliary initial boundary value problems in the left of a front crossection and in the right of a back crossection are investigated. The equivalency of flow problem with the obtained conditions and flow problem in infinitely long windtunnel is proved provided that flow is governed by linearized equations outside the computational domain. The questions of incorporating such conditions into difference schemes inside the computational domain are discussed.

A "Non-Overlapping" Composite Grid Scheme for the Solution of the Advection Equation in 2 Dimensions

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The use of composite grids can avoid the bottleneck of generating structured-grid for time-accurate solution of hyperbolic conservation laws in multiple dimensions. A fitted grid can be generated near a solid body fairly easily, while in the far field a Cartesian grid can be used. In contrast to the overlapping composite grid methods used by G. Chesshire, W. Henshaw, *et al*, we propose allowing the body-fitted grid patch to cut into the neighboring grid. With traditional explicit methods, the irregular small cells thus created would cause severe time-step restrictions. This restriction can be overcome by adapting the idea of h-boxes ($h = \Delta x$), as used by M. Berger and R. LeVeque on purely Cartesian grids, for use on composite grids. This approach has been analyzed in one space dimension and shown to yield high order methods on arbitrary grids. A composite grid method for the 2D advection equations has been developed based on a high-resolution wave propagation method for the advection equation. This approach should extend to systems of conservation laws such as the Euler equations as well.

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A Level Set Approach for Computing Solutions to Incompressible Two-Phase Flow

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Presented by Mark Sussman

A level set approach for computing solutions to incompressible two-phase flow is presented. The interface between the two fluids is considered to be sharp and is described as the zero-level set of a smooth function. We use knowledge from computing nonlinear PDE's in order to efficiently maintain, for all time and without explicitly finding or reconstructing the interface, the level set function as the signed distance from the interface. Consequently, we are able to handle arbitrarily complex topologies, large density and viscosity ratios, and surface tension, on relatively coarse grids. We use a second order projection method along with a second order upwinded procedure for advecting the momentum and level set equations. We consider the motion of air bubbles and water drops.

Stability of Viscous Rarefaction Waves

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I will discuss joint work with Kevin Zumbrun, Indiana Univ. on the time asymptotic stability of weak rarefaction waves for viscous strictly hyperbolic systems in one dimension. We treat localized perturbations with small mass and use pointwise estimates which are inspired by the work of Liu and Zumbrun on under-compressive shocks. The estimates are based on integral representations derived by approximate Green's functions and careful use of the Hopf-Cole transformation.

One aspect of the study is that the perturbed mass, compared to an approximate viscous "Burgers" rarefaction wave, is growing logarithmically with time t. As a consequence, the amplitude of the perturbation in a linear degenerate transversal field is $O(t^{-1/2} \log t)$ while it is $O(t^{-1/2})$ in a genuinely nonlinear transversal field, which is also an important ingredient in the stability proof.

Micro Void Production in RTM

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Presented by F.M. Tangerman

Micro void reduction is a major issue in the construction of reliable light weight components through Resin Transfer Molding (RTM). Since the amount of micro voids produced strongly depends on process conditions and on resin front interactions during filling, mold and process design benefit from numerical simulations of the process. We developed unsaturated flow models for micro void production, which we simulated through front tracking techniques. We demonstrate their accuracy through comparison with experimental data.

The Big Bang
or
Have You Ever Seen An Explosion Without a Shock Wave Before?

Blake Temple

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I will discuss the issue of irreversibility in Einstein' 'heory of gravitation as it relates to our recent paper *Shock ve Solutions of the Einstein Equations: The Oppenheimer-Snyder Model of Gravitational Collapse Extended to the Case of Non-Zero Pressure*, all of which is joint work with J. Smoller. In this paper we explicitly construct spherically symmetric shock wave solutions of the Einstein gravitational field equations for a perfect fluid by matching the Robertson-Walker (R-W) to the Interior Schwarzschild (I-S) metric at shock wave interfaces across which the gravitational metric is Lipschitz continuous. The R-W metric is a uniformly expanding solution of the Einstein equations which is generally accepted as a model for the universe at large; and the I-S solution is a time-independent solution which models the interior of a star. Both metrics are spherically symmetric in a radial variable, and both are determined by a system of ODEs that close when an equation of state for the fluid is specified. In our dynamically matched solution, we imagine the R-W metric as an exploding inner core (of a star or the universe as a whole), and the boundary of this inner core is a shock surface that is driven by the expansion behind the shock into the static I-S solution, which we imagine as the outer layers of a star, or the outer regions of universe. Our solution solves the problem first posed by Oppenheimer-Synder (O-S) in 1939 of extending their solution to the case of non-zero pressure. The O-S model is obtained by matching the R-W metric to the *empty* space Schwarzschild metric, and since in this case mass and momentum cannot cross the interface, (which in this case models the surface of a star), and keep the outer solution empty, O-S must make the well known unphysical assumption that the pressure be identically zero inside the star. In the classical theory of shock waves, the interface of O-S solution is a contact discontinuity, and this means that the solution is time-irreversible. In contrast, our shock-wave solution in which $p \neq 0$ is an irreversible solution of the Einstein gravitational field equations in which the irreversibility, loss of information, and increase of entropy in the fluids puts irreversibility into the dynamics of the gravitational field.

Linearized Asymptotic Inversion in the Presence of Caustics

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Presented by A.P.E. ten Kroode

In modern high frequency inversion of acoustic scattering data, standard ray theoretic methods play an important role, since the inversion operation is usually formulated in terms of traveltimes and amplitude functions computed along the rays. In this approach one formulates the inversion procedure in terms of a Fourier-integral operator acting on the scattering data. It is well known that this Fourier-integral operator reproduces the most singular part of the scattering potential, provided that there are no multiple ray paths between any two points in the medium. This rules out many practical situations in which caustics in the ray field are present.

We show that the inversion technique mentioned above can be extended to allow for caustics. The key is to replace standard ray theoretic expansions, which break down in caustic points, by uniform asymptotic expansions. The operator $\mathcal{F}$ relating high-frequency scattering data to the scattering potential is still a Fourier-integral operator in this case. Its invertibility boils down to the behaviour of the operator $\mathcal{F}^*\mathcal{F}$. We show that this operator can be written as the sum of a pseudo-differential operator and a non-local Fourier-integral operator and establish that the non-local part is harmless in a functional analytical sense. We also give explicit inversion operators.

On Integro-Differential Hyperbolic Systems Modelling Free Boundary Shear Flows

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Integro-differential systems of the form

$$U_t + A < U_x >= 0 \qquad (U = U(x, t, \lambda))$$

arise as a long wave approximation in modelling plane wave motions of vortical ideal fluid of finite depth under gravity. Here A is the special linear integral operator acting nonlocally with respect to Lagrangian variable λ (A depends on U). We develop mathematical theory for problems involving the systems with operator coefficients based on the generalizations of the hyperbolicity concept and methods applied in the theory of hyperbolic systems. The discrete and continuous spectrum of characteristic velocities and hyperbolicity conditions are obtained for both equations of incompressible and compressible barotropic flows. The local correctness of Cauchy problem for these systems is established when initial data satisfy hyperbolicity conditions.
The divergence form of the systems determining discontinuous solutions is proposed and properties of jump conditions are studied. Jump conditions include differential relations between flow parameters at both sides of discontinuity. Admissible shocks and simple waves are classified. The statement of the Riemann problem is discussed. This model describes new types of hydraulic jump on the flow with nontrivial vertical structure of horizontal velocity fields.

A Combined Adaptive Structured/Unstructured Technique for Essentially Unsteady Shock-Obstacle Interactions at High Reynolds Numbers

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Presented by Eugene Timofeev

The investigation of shock wave interactions with obstacles (diffraction, reflection etc.) is of fundamental and practical interest. For high Reynolds numbers the Eulerian (inviscid, non-heat-conducting) gas model is widely used providing reasonable results. Because the total number of mesh points is typically determined by the desirable resolution at discontinuities, dynamic adaptation of the grid to the solution and high-accurate methods might save considerably computer memory and CPU time consumption. The respective numerical technique based on adaptive unstructured triangular grids and high resolution Godunov-type scheme has been previously developed by the authors. The numerical results [1-4] have demonstrated a high accuracy, efficiency and reliability of the method for a wide range of transient shocked problems. However, for separated flows it fails to predict properly the location of separation points, the gasdynamic parameters on the body surface and surrounding wave pattern. The present study is aimed to extension of the above approach in such a way as to take into account viscous forces, primarily near solid surfaces.
The full Navier-Stokes equations written in an integral form with respective boundary and initial conditions underlie the mathematical model. To approximate adequately viscous terms near solid wall at high Reynolds number, thin layer (the thickness depends on Reynolds number) of non-uniform structured rectangular grid is generated along body surface. The rest of the computational domain is covered by an unstructured triangular grid. In the course of computation the unstructured grid is adapted to the solution using hierarchical refinement/unrefinement procedure. The structured grid remains fixed.
For both grids we establish nonoverlapping control volumes. The numerical methods are formulated in terms of fluxes through the volume faces. An unsplit explicit technique is employed on unstructured grid for time step operator. The inviscid (convective) terms are approximated by high-order Godunov-type scheme [2-4] while central differences are applied to viscous terms.
For structured grid the time step operator is split according to physical processes. To compensate partially severe limitation of the time step the inviscid operator is, in turn, split in space. The respective one-dimensional operators are approximated by the 1D version of the above Godunov-type scheme. The viscous operator is unsplit and contains central differences written implicitly in time. The implicit approximation is employed only for terms with second derivatives along normal direction to the body surface. The resulting set of linear difference equations with block tridiagonal matrix of coefficients is solved by the block Thomas algorithm. A predictor-corrector procedure allows to enhance temporal accuracy for all operators.

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The above approach has been applied to the computation of shock wave ($M_s = 3.0$) diffraction over a circular cylinder in CO_2 ($\gamma = 1.289$), the Reynolds number is about 10^5. Comparison between experimental data and computational results based on Euler and Navier-Stokes models is performed showing a good quality of viscous computations which describe correctly the development of separation and the location of separation points

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Two and Three Dimensional WAF-Type Finite Volume Schemes

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Presented by E. F. Toro

We present finite volume schemes for two and three dimensional systems of non-linear hyperbolic conservation laws. These are based on intercell numerical flux

$$F_{i+\frac{1}{2},j,k} = \frac{1}{\Delta t}\frac{1}{V(I)}\int_0^{\Delta t}\int_I F(U^*)dxdydzdt \qquad (1)$$

where U^* is the solution of local initial value problems, Δt is a time step, I is a spatial integration domain and $V(I)$ is its volume. Particular choices of U^*, Δt, I, and integral-evaluation schemes produce generalizations of familiar numerical methods, such as the Lax-Wendroff and Warming-Beam schemes. In this paper we first study schemes for the two and three dimensional linear advection model equation and then present two possible ways of extending one of these schemes to multi-dimensional non-linear systems.

First we derive schemes on the model equation that result from choosing (i) Δt consistent with a Courant number unity assumption, (ii) U^* is the solution of the multi-dimensional Riemann problem and (iii) I is the box surrounding the relevant intercell boundary and whose quadrilateral faces pass through the center of each neighbouring cell. Various integration rules are then applied, including exact integration. Several numerical schemes of first and second order accuracy with good stability properties are obtained. Strategies for oscillation-free versions of the schemes are discussed. Then we select a particular scheme and put forward two ways of extending it to multi-dimensional non-linear systems. We illustrate the ideas on the time-dependent, two-dimensional non-linear shallow water equations. Fig. 8 shows a sample numerical solution of a Mach reflection problem in shallow water

Generalized Roe's Solver for a Two-Fluid Model

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The model considered here is a first order equal pressure six equation two-fluid model. If we take the exception of the interfacial pressure term, which contains partial derivatives, the other terms of mass and momentum transfer between phases are given by algebraic relationships. They will appear as source terms of the model. The resulting model is a nonconservative hyperbolic one, consisting in mass, momentum and energy balance equations for each k-phase.

$$\begin{aligned}
\partial_t(\alpha_k\rho_k) + \partial_x(\alpha_k\rho_k u_k) &= 0 \\
\partial_t(\alpha_k\rho_k u_k) + \partial_x(\alpha_k\rho_k u_k^2) + \alpha_k\partial_x p + (p - p_i)\partial_x\alpha_k &= 0 \\
\partial_t(\alpha_k\rho_k E_k + p\partial_t\alpha_k + \partial_x(\alpha_k\rho_k u_k E_k + \alpha_k p u_k) &= 0
\end{aligned}$$

The most common approach solving these equations is numerical method based on staggered grids and donor-cell differencing [1]. This method, now almost universal in two-fluid computer code introduces a large amount of numerical diffusion. Moreover, high frequency oscillations may appear with a large number of cells. The nonconvergence of the results may be due either to the ill-posed character of the model or to the numerical scheme.

Here, we present a numerical method based upon a generalized Roe's approximate Riemann solver. Such numerical schemes are widely used for hyperbolic systems of conservation laws. The linearized approximate Riemann solver of Roe [2] was proposed for the numerical solution of the Euler equations governing the flow of an ideal gas. A weak formulation of Roe's approximate Riemann solver, based on the choice of a path Φ in the state space, has been introduced in [3] to cope with arbitrary equation of states. This weak formulation was applied in order to build a Roe-averaged matrix for a conservative system governing a homogeneous equilibrium two-phase flow. Here, we extend this method to a hyperbolic nonconservative system that models a two-component two-phase flow.

The construction of this approximate Riemann solver uses a linearization of the nonconservative products in the system (1), and an extension of Roe's method. Precisely, we consider approximate solutions to the Riemann problem for the system (1) which are exact solutions to the approximate linear problem:

$$\partial_t \mathbf{u} + A(u_L, u_R)_\Phi \partial_x \mathbf{u} = 0$$
$$\mathbf{u}(x,0) = \mathbf{u}_L(x < 0), \quad \mathbf{u}(x,0) = \mathbf{u}_R(x > 0)$$

where $A(u_L, u_R)_\Phi$ is a constant matrix satisfying a generalized jump condition:

$$A(u_L, u_R)_\Phi(u_R - u_L) = \int_0^1 A(\Phi(s, u_L, u_R))\frac{\partial\Phi}{\partial s}(s, u_L, u_R)\,ds$$

To construct such matrix we follow the method introduced in [3], where the main feature is the choice of the canonical path for a parameter vector w:

$$\Phi(s, u_L, u_R) = \psi_0(w_L + s(w_R - w_L))$$

with the parameter vector chosen as follow

$$w = (\sqrt{\rho c}, \sqrt{\rho(1-c)}, \sqrt{\rho c}u_v, \sqrt{\rho(1-c)}u_l, \sqrt{\rho c}H_v, \sqrt{\rho(1-c)}H_l).$$

In practice this new numerical method has proved to be robust and capable of generating accurate non-oscillating solutions for two-phase flow calculations. The scheme was applied both to shock tube problems and to standard test for two-fluid computer codes.

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The Global Existence Theorem for Gas Dynamics System

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The one dimensional system (consisting of mass, impulse and energy conservation laws) is considered. The global existence theorem of functional solution is proved for Cauchy problem in this case. The initial date are arbitrary locally bounded functions. So, any finite value and types of discontinuity for initial date are possible. The previous article [1] of Liu Tai-Ping connected with above mentioned problem is more restrictive (the small variation of initial date is supposed). The global solution is limit of weakly convergent approximations of viscosity method. The approximations belong to the space of locally summable functions. The background of global existence theorem is based on functional solutions theory [2] produced by V.A. Galkin.

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Convergence to Equilibrium for a System of Conservation Laws with a Relaxation Term

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Presented by Aslak Tveito

Conservation laws with relaxation terms appear in a series of important applications, cf. [1,2,3] and references given therein. We analyze a system of conservation laws of the following form,

$$
\begin{aligned}
(u+v)_t + f(u)_x &= 0 \\
\delta v_t &= A(u) - v.
\end{aligned} \tag{0.1}
$$

Here f and A are given functions, u and v are the unknowns and $\delta > 0$ is referred to as the relaxation time. The function $A(u)$ is an increasing function of u. A physical motivation for this problem is discussed in [3].
Formally, by letting δ tend to zero in (0.1), we get a scalar conservation law of the following form

$$(w + A(w))_t + f(w)_x = 0. \tag{0.2}$$

The purpose of this talk is to present the results derived in [2] concerning the convergence of the solutions of the system (0.1) towards the solutions of the equation (0.2) as δ tends to zero.

THEOREM 0.1. *Let (u^0, v^0) be a pair of initial data satisfying i) $(u^0, v^0)(x) \in [0,1] \times [0,1]$ for all x, ii) $(u^0, v^0) \in BV$, and iii) $\|A(u^0) - v^0\|_1 \leq M\delta$. Furthermore, the initial condition for the scalar equation (0.2) is given by $w^0 = u^0$. Then, for any $T > 0$, there is a finite constant M such that*

$$\|u(\cdot,t) - w(\cdot,t)\|_1 \leq M\delta^{1/3} \quad \text{for all } 0 \leq t \leq T.$$

Here (u,v) and w are solutions of (0.1) and (0.2) respectively. □

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Uses of Preconditioned Euler Equations

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Since 1990 it has been known that local preconditioning of the Euler equations of the form

$$\frac{\partial U}{\partial t} + P\left(A(U)\frac{\partial U}{\partial x} + B(U)\frac{\partial U}{\partial y} + C(U)\frac{\partial U}{\partial z}\right) = 0,$$

where $P(U)$ is the preconditioning matrix, can make the condition number of the characteristic speeds, i. e., the eigenvalues of the matrix $PAn_x + PBn_y + PCn_z$, where n is a unit vector, as low as $1/\sqrt{1 - \min(M^2, M^{-2})}$, down from the standard value $(M + 1)/\min(M, |M - 1|)$. This result has not previously been reported at any of the conferences on Hyperbolic Problems.

There are a number of numerical benefits to the use of such a preconditioning:

1. It removes the stiffness of the system of Euler equations caused by the range of wave speeds, thus improving the rate of convergence of any marching scheme to a steady solution.

2. It makes the system of equations behave more like a scalar equation, which simplifies the design and analysis of any auxiliary numerical technique.

3. It may prevent the loss of spatial accuracy for $M \to 0$, observed in the use of standard Euler schemes.

Examples of each of these three uses will be presented.

Convergence of the Bidimensional Nessyahu-Tadmor's Scheme on a Non-structured Mesh for a Linear Hyperbolic Equation

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Presented by M.-C. Viallon

The scheme of Nessyahu-Tadmor in one spatial dimension is a second order scheme of Van Leer type, using the approximate Riemann solver of the Lax- Friedrichs scheme. The convergence is proved in the scalar case[1]. Here, we consider the bidimensional version of this scheme. We use a triangular non-structured meshe of R × R to solve :

$$\begin{aligned} \tfrac{du}{dt} + V\,grad(u) &= 0, in\, \mathbb{R}\times\mathbb{R} \\ u(x,0) &= u_0(x) \end{aligned} \qquad (1)$$

The scheme consists in two steps. We consider the approximate solution $u(t_n)$; then using a finite volume formulation on the barycentric cells C_i , constructed around the vertices i of the triangles, leads, upon integrating the equation in C_i, to the approximate solution $u(t_{n+1})$. Then integrating a second times in a "sub-barycentric cell", we obtain $u(t_{n+2})$. We establish a weak estimate on the spatial derivatives which ensures the weak star convergence in the space of essentially bounded functions to the weak solution of the linear problem.

[1] Nessyahu, H and E. Tadmor, "Non-Oscillatory Central Differencing for Hyperbolic Conservation Laws," *J. Comp. Phys.*, vol. 87, pp. 408–463, 1990.

Similarity Reductions and Painleve Analysis for Nonlinear Boltzmann Equation

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Presented by K. Vijayakumar

Using scale invariant transformation we have obtained a new class of exact solution to the Boltzmann equation of order $(p+1)$. Further, computation of symmetry operators for Boltzmann equations of order 2,3,4 and 5 has enabled us to conjecture the symmetry operator for all $p \in I^+$ which leads to Lie Algebra of order 4. For $p = 1$, which corresponds to the Krook-Wu model of the Boltzmann equation, the resulting equation has been studied for group invariant solutions through similarity reduction and Painleve analysis. It is interesting to point out that Painleve analysis has, beside providing three new exact solutions to the Boltzmann equation, enabled us to recover two well known solutions available earlier in the literature.

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Conservation Laws and Exterior Calculus

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The natural language for divergence-form conservation laws is the language of differential forms. Use of differential forms helps us to recognize facts that have not previously been readily apparent. In particular, such use shows quite clearly that equations of continuity for solid mechanics should be transformed (under coordinate transformations) as differential forms of degree one, and not as differential forms of degree n-1. In addition we show that weakly closed differential forms transform, under locally bi-Lipschitz coordinate transformations, into weakly closed forms, and that exterior products of weakly closed forms are weakly closed. This allows us to resolve a mystery in solid mechanics regarding redundant conservation laws for the deformation gradient.

3D Simulations of Colliding Hypersonic Radiative Flows in Astrophysics

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Simulations of colliding hypersonic radiative flows are presented. For the simulations we numerically solved the inhomogeneous Euler equations with a multidimensional high resolution integrator of the MUSCL type [2]. To achieve the necessary spatial resolution we made use of the adaptive mesh refinement algorithm of Berger [1].

Jets, supernova remnants, stellar winds, and accretion flows are examples for hypersonic flows in astrophysics. The Mach Numbers can easily reach several hundred. A common feature of the flows is the collision with surrounding matter, e.g. the interstellar medium or a second hypersonic flow. Complex flow patterns evolve showing interaction of shocks, instabilities of different types and turbulence (see Figure 9). This collision greatly influences the observable features of the objects, e.g., the spectra or its shape. Therefore, a proper simulation of such collision zones is essential.

The numerical modeling of hypersonic colliding flows is very demanding for several reasons. We find strong shock waves interacting with each other. The resulting high temperature regions are effectively cooled by radiation. This process leads to thermal instabilities which are difficult to compute because the thermal cooling time scale may be many orders of magnitude smaller than the dynamical time scale of the flow. In double star systems, the two stellar components, which are the sources of the hypersonic flows, move in ellipses around each other. Therefore, outflow boundary conditions from moving objects have to be implemented in the calculations.

For the first time, collision of hypersonic astrophysical flows can be computed properly in 3D using advanced numerical techniques such as high resolution integrators and adaptive mesh refinement. Another important aspect of this work is the visualization of the 3D flow pattern.

[1] M.J. Berger, "Adaptive Mesh Refinement for Hyperbolic Equations," *Lectures in Applied Mathematics*, vol. 22, pp. 31–40, 1985.

[2] P. Colella, "Multidimensional Upwind Methods for Hyperbolic Conservation Laws," *J. Comput. Phys.*, vol. 87, pp. 171, 1990.

Models for Rate Dependent Plasticity

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The purpose of this talk is to analyze an intrinsic limitation on strain rate in the widely used Steinburg-Lund model for rate dependent plasticity and to propose a modification which removes this limitation and allows a fit to experimental data with strain rate up to $10^6\ \mathrm{sec}^{-1}$. The Steinberg-Lund model, when inverted to give an inverse rate, or characteristic time, is a sum of two terms. The first represent the time for a dislocation to overcome a potentialbattier to its motion. The second is viscous drag, for the motion of a dislocation across the valleys between the barriers. The proposed modification is to remove the effect of the first term smoothly at high strain rates, as the barriers are blow available strain energies in this sense

Homogenization in Elastodynamics with Force Term Depending on Time

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In this short work we extend the study on the homogenization problem for an elastic material containing a distributed array of gas bubbles to the case when the body force depending on time. By technically constructing an approximating sequence, we are able to show the convergence of semigroups and therefore prove the main result that such spongy material can be approximated by a uniform elastic material in elastodynamics for general force term.

Superconvergence of Mixed Finite Element Methods for Maxwell's Equations

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Presented by J.C. Wood

For the numerical solution of Maxwell's equations, finite element methods applied to the differential forms are becoming viable contenders to the historically older integral equation finite element techniques. Unfortunately, for the majority of finite element methods in use, little, if any, analysis is available. We consider the stability and accuracy properties of two classes of mixed finite element methods for the time-dependent Maxwell system. The first, known as the ECHL scheme, uses piecewise constant basis functions to approximate the electric field and continuous bilinear basis functions for the magnetic field. The second popular family of methods, developed by Nédélec, employs a hierarchy of curl-conforming basis functions to discretize the electric field and standard finite element basis functions for the magnetic field.

Using energy analysis, we present a complete stability and error analysis for these methods. We derive optimal error bounds and establish superconvergence properties at points associated with the degrees of freedom and with respect to carefully selected discrete L_2-norms. We develop new techniques for bounding the truncation errors of these schemes, and admit quite general assumptions on the boundary conditions

[a]This work has been supported by the Science and Engineering Research Council (SERC).

On Mode III Propagation of Adiabatic Shear Bands

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Presented by T.W. Wright

Adiabatic shear bands are severe localizations in plastic deformation due to a material instability. As plastic deformation occurs, the material heats up due to plastic work. When thermal softening overcomes work hardening and rate hardening combined, strain softening begins and a local instability has a chance to occur. In one dimension, as in a Hopkinson bar torsion test, the material loses strength in a catastrophic manner, but in two dimensions a contact discontinuity develops and propagates edgewise, somewhat like a crack in a brittle material. Instead of a stress free surface, however, the shear band has a sliding discontinuity with large heat generation. This paper begins a discussion of the mechanics of such surfaces, the simplest case being the mode III of antiplane motion. In analogy to the crack problem, a similarity solution for the region close to the end of the shear band is found, but here the singularity is much weaker and depends on the strain rate sensitivity. As a consequence, mathematically the tip of a shear band seems to be rather diffuse, which also seems to correspond to the physical situation.

Nonlinear Stability of Contact Discontinuity

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The stability of contact discontinuities of the full 1-D compressible Euler equations is subtle. The question is whether a contact discontinuity can be an asymptotic state for the corresponding viscous system. In this talk I will present a recent result which shows that contact discontinuities are "metastable" wave patterns for the compressible Euler equations for ideal gases with uniform viscosity. More precisely, it is found that although a contact discontinuity is not an asymptotic attractor for the viscous system, yet a viscous wave pattern, which approximates the contact discontinuity on any finite time interval, can be constructed, and which is shown to be asymptotically stable with small generic perturbations for the viscous system provided that the strength of the contact discontinuity is suitably small. In fact, a detailed asymptotic form of the viscous solution can be obtained. A generic perturbation of the contact discontinuity introduces a shift in the center of the diffusive contact wave, and nonlinear diffusion waves in the two sound wave families. This asymptotic form can be determined a priori by the distribution of the initial excessive mass. A coupled linear diffusion wave located essentially in the region of the diffusive contact wave is needed to obtain the asymptotic ansatz. The stability is obtained by using a carefully designed weighted energy estimate. This is joint work with D. Hoff.

Gevrey Well-Posedness of Nonstrict Hyperbolic Equations

S. Ya. Yakubov and Ya. S. Yakubov [a]

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School of Mathematical Sciences, Tel-Aviv University, Israel

Presented by Ya. S. Yakubov

Consider in principal an initial boundary value problem for a nonstrict hyperbolic equation

$$\frac{\partial^{2n}u(t,x)}{\partial t^{2n}} + \sum_{k=1}^{n} \frac{\partial^{2(n-k)}}{\partial t^{2(n-k)}} \left(a_k(x)\frac{\partial^{2k}u(t,x)}{\partial x^{2k}} + B_k u(t,\cdot)|_x \right) = 0, \quad (1)$$

$$\sum_{k=0}^{n_\nu} \frac{\partial^{2k}}{\partial t^{2k}} \left(\alpha_{\nu k}\frac{\partial^{2(n_\nu-k)}u(t,0)}{\partial x^{2(n_\nu-k)}} + \beta_{\nu k}\frac{\partial^{2(n_\nu-k)}u(t,1)}{\partial x^{2(n_\nu-k)}} + T_{\nu k}u(t,\cdot) \right) = 0, \quad (2)$$

$$\frac{\partial^k u(0,x)}{\partial t^k} = u_{k+1}(x), \quad k = 0,\ldots,(2n-1), \quad (3)$$

where $(t,x) \in R \times [0,1]$, $\nu = 1,\ldots,2n$, integers $n \geq 1$, $n_\nu \leq n-1$; $\alpha_{\nu k}$ and $\beta_{\nu k}$ are complex numbers, $|\alpha_{\nu 0}| + |\beta_{\nu 0}| \neq 0$; $a_k(\cdot) \in C[0,1]$, $a_n(x) \neq 0$ $\forall x \in [0,1]$, $a_k(0) = a_k(1)$; roots $\omega_1(x),\ldots,\omega_{2n}(x)$ of the characteristic equation $a_n(x)\omega^{2n} + a_{n-1}(x)\omega^{2(n-1)} + \cdots + a_1(x)\omega^2 + 1 = 0$ are real $\forall x \in [0,1]$ and the roots $\omega_1(x),\ldots,\omega_n(x)$ are situated on the left side of the imaginary axis and the roots $\omega_{n+1}(x),\ldots,\omega_{2n}(x)$ on the right side; the system $\alpha_{\nu 0}u^{(2n_\nu)}(0) + \beta_{\nu 0}u^{(2n_\nu)}(1) + T_{\nu 0}u$, $\nu = 1,\ldots,2n$, is normal. We introduce some n-component functional Banach space E and associate with problem (1)–(3) some operator A in E and corresponding Gevrey space $G(\alpha;A)$ which depends on a parameter α.

Theorem. *Let the following conditions be satisfied:*

1. $\theta(0) \neq 0$, $\theta(1) \neq 0$, *where* $\theta(x) = \det\Theta(x)$ *and*

$$\Theta_{\nu\ell} = \begin{cases} \sum_{k=0}^{n_\nu} \alpha_{\nu k}\omega_\ell^{2(n_\nu-k)}, & \ell = 1,\ldots,n, \\ \sum_{k=0}^{n_\nu} \beta_{\nu k}\omega_\ell^{2(n_\nu-k)}, & \ell = n+1,\ldots,2n. \end{cases}$$

   for $\nu = 1,\ldots,2n$.

2. *linear operators* B_k *from* $W_q^{2k}(0,1)$ *into* $L_q(0,1)$ *are compact,* $\exists q \in (1,+\infty)$;

3. *linear functionals* $T_{\nu k}$ *are continuous in* $W_r^{2(n_\nu-k)}(0,1)$, $\exists r \in [1,+\infty)$;

4. $(u_1(\cdot),u_3(\cdot),\ldots,u_{2n-1}(\cdot))$, $(u_2(\cdot),u_4(\cdot),\ldots,u_{2n}(\cdot))$ *belong to* $G(\alpha;A)$, *where* $0 < \alpha < 2$.

Then $G(\alpha;A)$ *is dense in* E *and problem* (1)–(3) *has a unique strong solution* u *such that* $(u(t,\cdot),u''(t,\cdot),\ldots,u^{(2n-2)}(t,\cdot))$ *belongs to* $G(\alpha;A)$ *for all* $t \in R$.

[a] **Both authors were supported by the Israel Ministry of Absorption.**

The Method of Wavewise Entropy Inequalities for Numerical Analysis of Hyperbolic Conservation Laws

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In this talk, we introduce the method of wavewise entropy inequalities for analysis of entropy consistency of TVD non-oscillatory shock capturing schemes for Cauchy problems of conservation laws

$$u_t + f(u)_x = 0$$

Let u_j^n be the numerical solution, $g_{j+1/2}$ be the numerical flux. and $\bar{u}_j = (u_j^n + u_j^{n+1})/2$. For convex flux f, we present the following criterion for entropy consistency: Suppose $f^{LIN}(x)$ is the linear function with $f^{LIN}(a) = f(a)$ and $f^{LIN}(b) = f(b)$ such that $f^{LIN}(x) > f(x)$ for $a < x < b$. Then the scheme is entropy consistent if the following quadrature inequality holds. For any such a and b, there is a positive constant δ such that if $a = \bar{u}_I \leq \bar{u}_{I+1} \leq \cdots \leq \bar{u}_J = b$ and if $g_{I-1/2} = g_{I+1/2}$ and $g_{J-1/2} = g_{J+1/2}$, then

$$\sum_{j=I}^{J-1} (\bar{u}_{j+1} - \bar{u}_j)g_{j+1/2} + \delta < \int_a^b f^{LIN}(x)dx$$

Examples of both semi-discrete and fully-discrete schemes, of both Godunov type and the type using flux limiters are presented to show the applications of the criterion. The convergence of these schemes has been open problems.

A Functional Integral Approach to Shock Wave Solutions of the Euler Equations with Spherical Symmetry

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In this paper, we consider the spherically symmetric Euler equations for compressible gas dynamics in R^3 outside a fixed ball. For $n \times n$ systems of conservation laws in one dimension without source terms, the existence of global weak solutions was proved by Glimm [1]. Glimm constructed approximate solutions using a difference scheme by solving a class of Riemann problems. It is well known that there is no proof for global existence of weak solutions for more than one dimension, although there are many results for one dimension. In our problem, the Euler equations can be reduced to a one dimensional problem by spherical symmetry, but there is a non-integrable source term, and it is an open problem to show that the solutions of these equations remain bounded for all time.

We consider the Cauchy problem for the Euler equations in the spherically symmetric case when the initial data are small perturbations of the trivial solution, i.e. $u \equiv 0$ and $\rho \equiv$ *constant*, where u is velocity and ρ is density. We show that this Cauchy problem can be reduced to an ideal nonlinear problem approximately. If we assume all the waves move at constant speeds in the ideal problem, by using Glimm's scheme and an integral approach to sum the contributions of the reflected waves that correspond to each path through the solution, we get uniform bounds on the L^∞ norm and total variation norm of the solutions for all time. The geometric effects of spherical symmetry lead to a non-integrable source term in the Euler equations. Correspondingly, we consider an infinite reflection problem and solve it by considering the cancellations between reflections of different orders in our ideal problem. Thus we view this as an analysis of the iteration effects at the quadratic level in a nonlinear model problem for the Euler equations. Although it is far more difficult to obtain estimates in the exact solutions of the Euler equations due to the problem of controlling the time at which the cancellations occur, our analysis leads to an interesting new norm for the initial data which is appropriate for proving the stability (in the total variation norm) of the solutions in the spherically symmetric setting. And we have identified a new global dissipative mechanism that is not based on the nonlinearity of the wave speeds. We believe that this analysis of the wave behaviour will be the first step in solving the problem of existence of global weak solutions for the spherically symmetric Euler equations outside a fixed ball.

Solutions to the Euler Equations with Large Data

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Presented by Robin Young

We consider the full 3×3 Euler equations for inviscid gas dynamics in one space dimension. We show that for any large time T, solutions to the Cauchy problem exist up to time T, for initial data with arbitrarily large bounded variation, as long as the sup-norm is small enough. As a corollary, we obtain a large-time existence result for periodic solutions. We obtain an explicit discretized "path integral" formula for a modified Glimm functional. We then find a time-dependent bound for this functional which is independent of the mesh size. This ensures that a subsequence of the Glimm approximants converges to a solution. The analysis relies heavily on the fact that the entropy is a Riemann coordinate, so that any new entropy generated by weak interactions is cubic in the strengths of incident waves. In general, some assumption on the system is needed to avoid blowup of solutions in finite time. On the other hand, our analysis does not take nonlinear decay into account. The inclusion of nonlinear decay should yield stronger results, possibly including time-independent bounds.

[a]This research was supported by a grant from the Dept. of Energy, Grant Number DE-FG02-88ER25053.

The Asymptotic and Transient Scalings of Fluid Mixing Induced by Random Fields

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We have developed a qualitative theory for the mixing of fluids induced by a random velocity field. The theory provides a quantitative prediction for the growth of the mixing region. There are three distinct regimes for the asymptotic scaling behavior of the mixing layer, depending on the asymptotic behavior of the random velocity field. The asymptotic diffusion is Fickian when the correlation function of the random field decays rapidly at large length scales. Otherwise the asymptotic diffusion is non-Fickian. The scaling behavior of the mixing layer in a general random velocity field is determined over all length scales. Our results show that, in general, the scaling exponent of the mixing layer is non-Fickian on all finite length scales. In the Lagrangian picture, due to the non-linearity of the effective dynamical equation derived from the Taylor diffusion theory, the mixing layer is not a fractal even if the random velocity field is a fractal.

Two-Dimensional Riemann Problems for Systems of Conservation Laws

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A certain type of Riemann problem for the two-dimensional Euler equations in gas dynamics is summarized.

1. The problem is ultimately classified from sixteen to eighteen cases.
2. Numerical simulation for each case has been done.
3. None of the cases has been completely treated analytically.

In order to approach it, a simplified model of a 2×2 system has been treated with analytical proof as well as numerical simulation. A new kind of nonlinear hyperbolic wave, the Delta-shock appears in some solutions.

[a] Research supported partly by NSFC.

Colloid Layering as a Dispersive Phenomenon

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Colloid suspensions exhibit layering behavior in experiment, which has not been completely explained mathematically. We discuss a model developed by Rubinstein to describe the evolution of a stratified suspension of particles in a fluid. Our analysis suggests that layering is a dispersive effect that is quite familiar in other contexts.

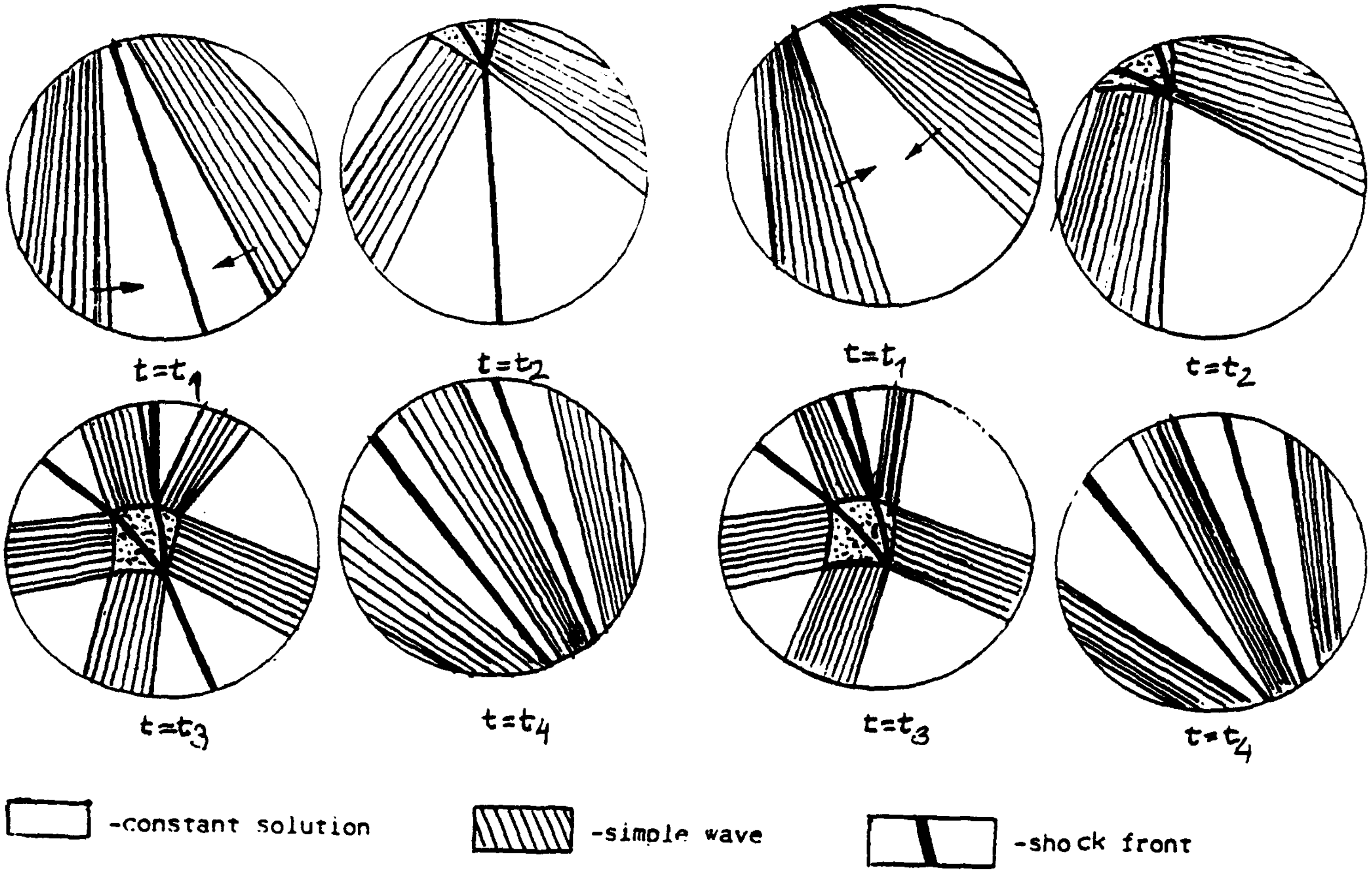

Figure 1: Two films of wave interactions.

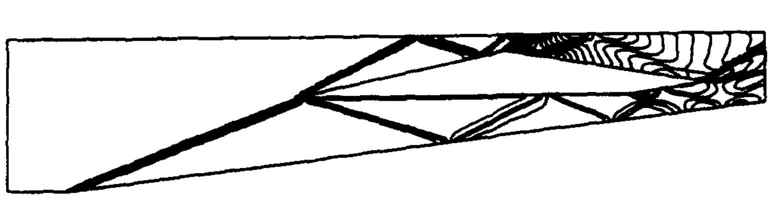

Figure 2: Flowfield in the inlet of a scram jet.

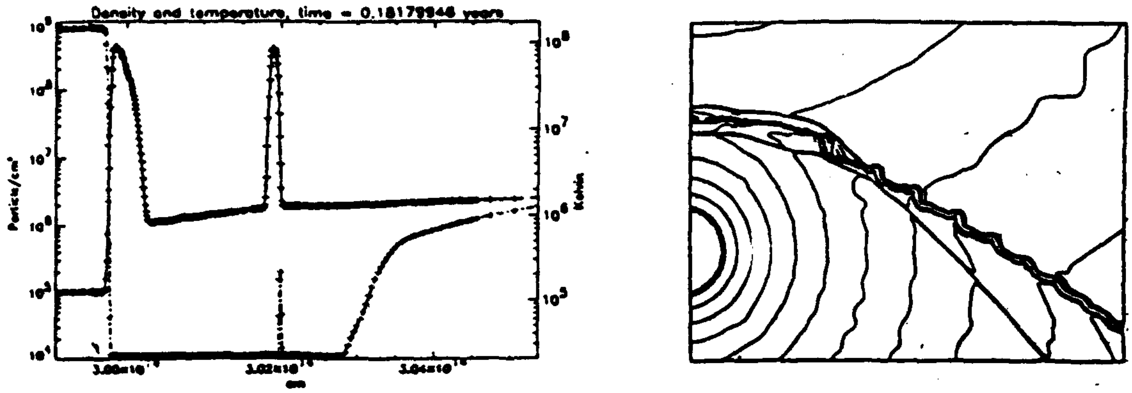

Figure 3: Complex structure of a radiative shock wave and the working mechanism of AMR in a 1D calculation. Cell boundaries are indicated by dashes (a). Dynamical instability of the cold gas layer demonstrated in a cylindrically symmetric computation (b).

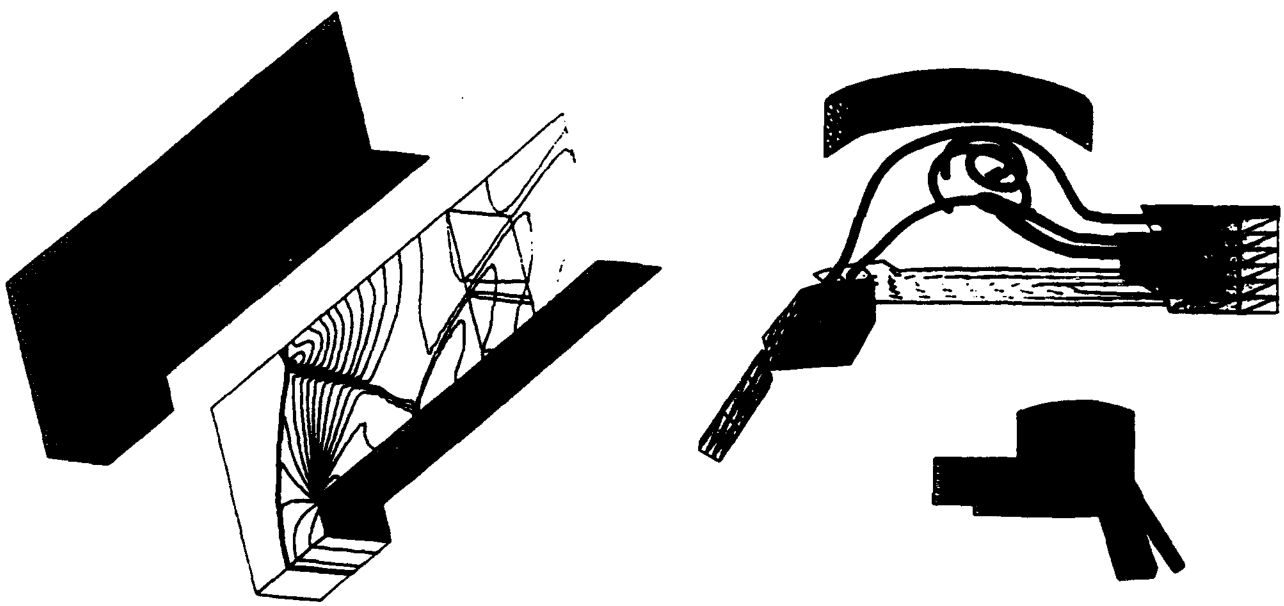

Figure 4: The flow in a complex cylindrical geometry with a moving piston.

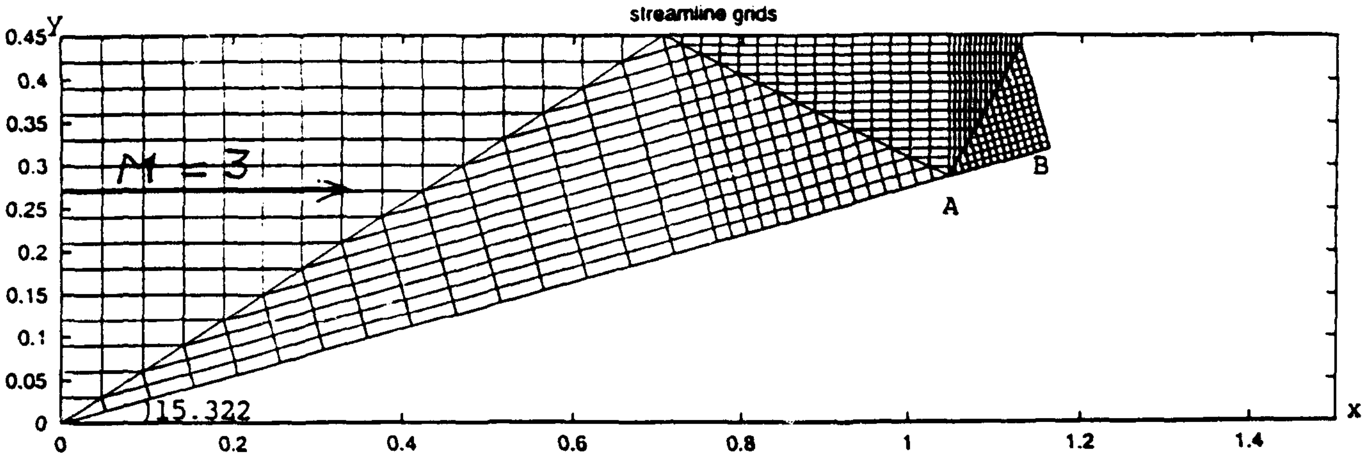

Figure 5: Computed grid and shocks using the optimal grid formulation.

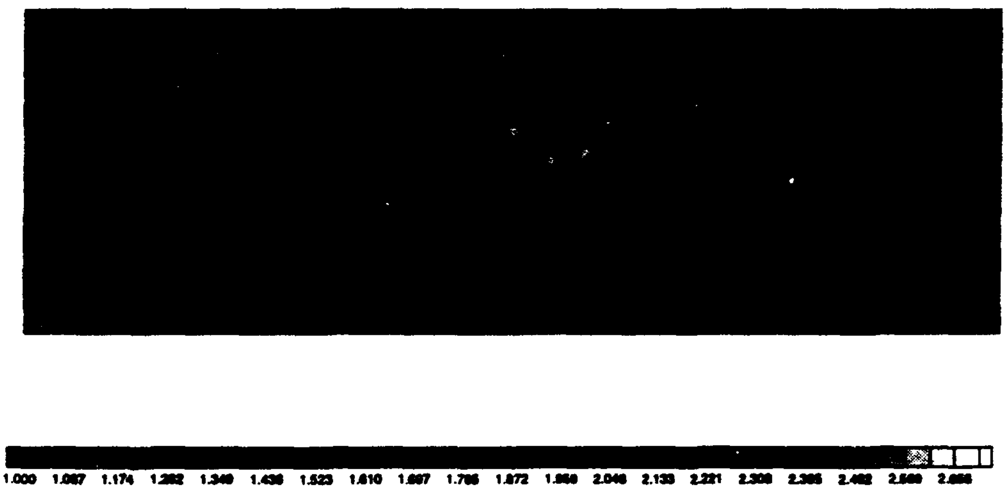

Figure 6: Flow over a bluff body. Flat plate, placed normal to the flow. Mach number at infinity: $M = 0.6$, reference Reynolds number (based on plate length) $Re_L = 250$, Prandtl number $Pr = 0.72$. Solved with Chebyshev spectral element flux-corrected method on a mesh consisting of 300 elements, with 9 Chebyshev-Gauss-Lobatto points per element and per coordinate direction. 40 iso-Mach contours.

AD-A283 078 INTERNATIONAL CONFERENCE ON HYPERBOLIC PROBLEMS THEORY NUMERICS APPLICATIONS HELD IN STONY BROOK NEW YORK ON 13-17 JUNE 1994(U) STATE UNIV OF NEW YORK AT STONY BROOK R ABGRALL ET AL. 25 JUL 94 XB-ONR 2/2

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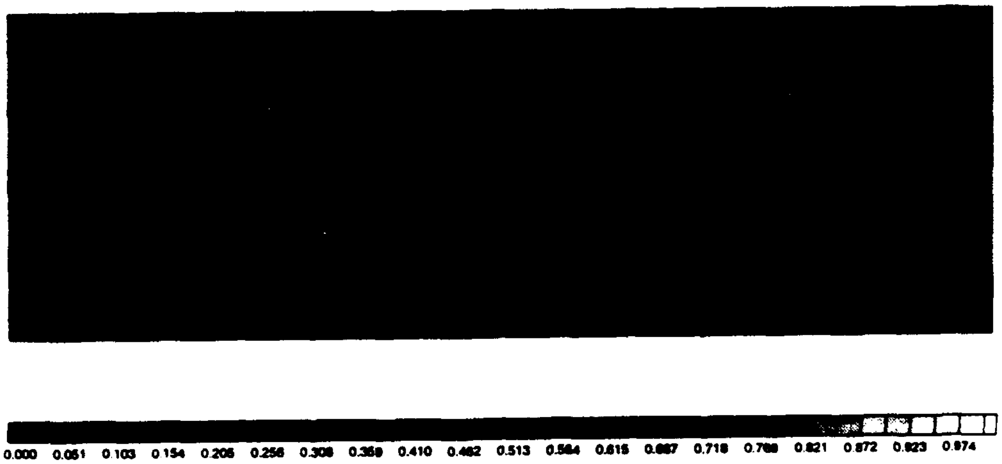

Figure 7: 40 contours of non-dimensionalized pressure.

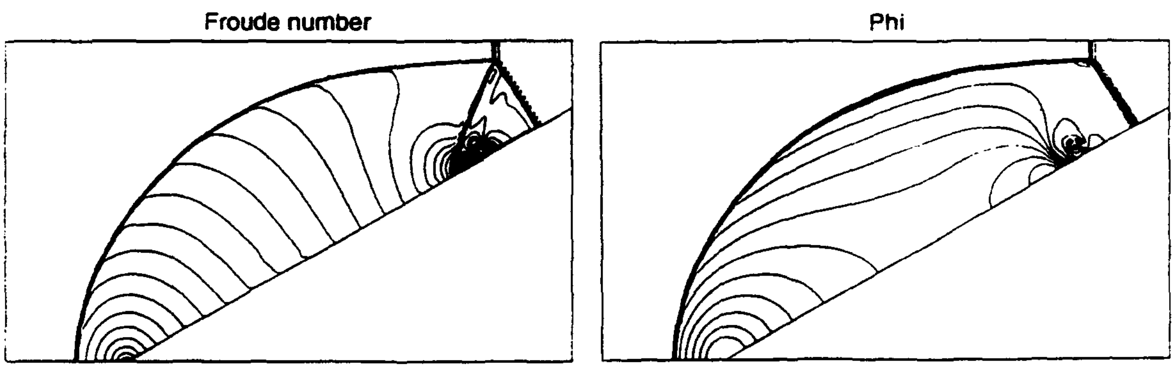

Figure 8: A sample numerical solution of a Mach reflection problem in shallow water.

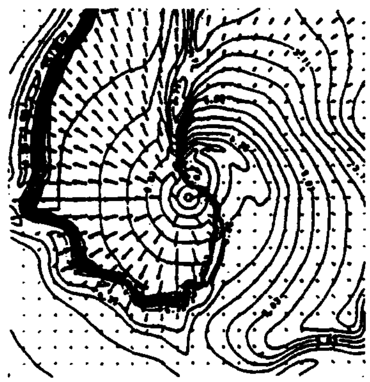

Figure 9: Flow pattern of colliding hypersonic winds in the double star system EG AND.